20.03.12

# Quantum Interference Effects



QM a la Feynman : Path integrals

[ R.P. Feynman, Rev Mod Phys **20**, 367 (1948); Nobel prize 1965 for contributions to quantum electrodynamics (QED) ]

$$\underbrace{\Psi(\vec{r},t)}_{\text{state at time } t>t'} = \int d\vec{r}\,'\; G(\vec{r},t;\vec{r}\,',t')\,\underbrace{\Psi(\vec{r}\,',t')}_{\text{state at time } t'<t}$$

Propagator (or: Green function):

$$G(\vec{r},t;\vec{r}\,',t') \sim \sum_{\substack{\uparrow \\ \text{all possible paths from } (\vec{r}\,',t') \text{ to } (\vec{r},t) \text{ in "space-time"}}} e^{i\,S(\vec{r},t;\vec{r}\,',t')/\hbar}$$

$$\equiv \int_{\vec{r}\,',t'}^{\vec{r},t} \mathcal{D}\vec{r}(\tau)\; e^{i\,S[\vec{r}(\tau)]/\hbar} \qquad (\text{= path integral})$$

where $S$ is the <u>classical action</u>:

$$S(\vec{r},t;\vec{r}\,',t') = \int_{t'}^{t} d\tau\, L = \int_{t'}^{t} d\tau\,(T-U)$$

with
- $L$ = Lagrangian
- $T$ = kinetic energy
- $U$ = potential energy

How is $S$ affected by a magnetic field, $\vec{B} = \nabla\times\vec{A}$, i.e.
what is $\Delta S = S(\vec{A}) - S(0)$ ?

$$L(\vec{A}=0) = T - qV = T + eV \qquad (\text{for electrons, } q=-e)$$

(*) $$L(\vec{A}) = T - qV + q\vec{v}\cdot\vec{A} = L(0) - e\vec{v}\cdot\vec{A}\;; \quad \vec{v} = d\vec{l}/d\tau$$

$$\Rightarrow \Delta S = -e\int_{t'}^{t} d\tau\,\vec{v}\cdot\vec{A} = -e\int_{\vec{r}\,'}^{\vec{r}} \vec{A}\cdot d\vec{l}$$

(*) See e.g. Goldstein, "Classical mechanics", or lecture notes TFY4345

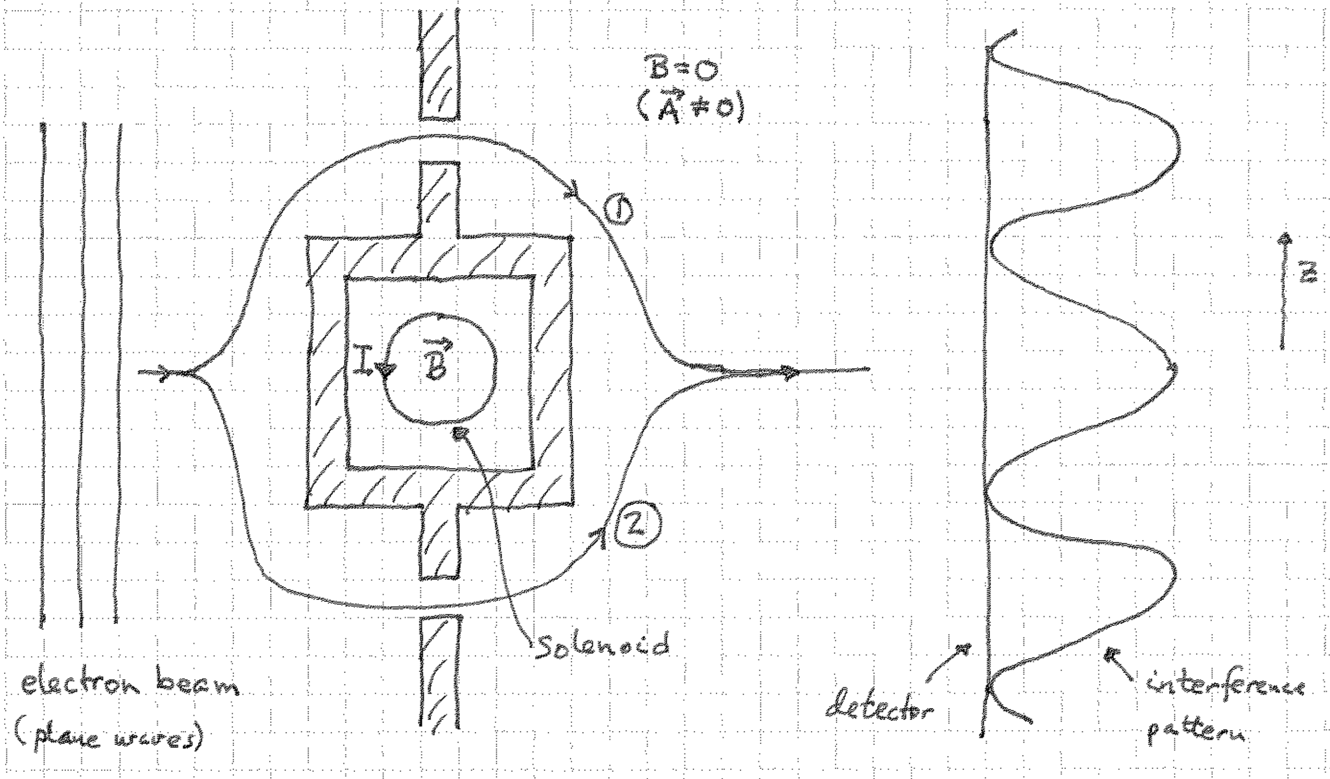
$$\rightarrow \quad G(\vec{r},t;\vec{r}',t') = \int_{\vec{r}',t'}^{\vec{r},t} \mathcal{D}\vec{r}(\tau)\, e^{\frac{i}{\hbar}S(\vec{r},t;\vec{r}',t';0)} \quad e^{-\frac{ie}{\hbar}\int_{\vec{r}'}^{\vec{r}} \vec{A}\cdot d\vec{l}}$$

[See also M.V. Berry, Proc. R. Soc. London A <u>392</u>, 45 (1984);
"Berry phase" in quantum mechanics]

[The Schrödinger eqn may be <u>derived</u> from Feyman's path integral approach.]

## Aharonov - Bohm effect

Original proposal: A&B, Phys Rev <u>115</u>, 485 (1959)




["ordinary" QM interference]

$\vec{B} \neq 0$ only inside solenoid

QM phase factors due to $\vec{B} = \nabla \times \vec{A}$:

along path ①: $\Delta\varphi_1 = (-e/\hbar)\int_{①} \vec{A}\cdot d\vec{l}$

along path ②: $\Delta\varphi_2 = (-e/\hbar)\int_{②} \vec{A}\cdot d\vec{l}$

$|\psi_1| \approx |\psi_2|$ (by symmetry)

$\Rightarrow$ phase difference between $\psi_1$ and $\psi_2$:

$$\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = \frac{e}{\hbar}\left\{ \int_{①} \vec{A}\cdot d\vec{l} - \int_{②} \vec{A}\cdot d\vec{l} \right\}$$

$$= \frac{e}{\hbar} \oint \vec{A}\cdot d\vec{l} \overset{\text{Stokes}}{=} \frac{e}{\hbar} \int (\nabla\times\vec{A})\cdot d\vec{S}$$

$$= \frac{e}{\hbar} \int \vec{B}\cdot d\vec{S} = \frac{e}{\hbar}\Phi$$

$\Phi = \int \vec{B}\cdot d\vec{S}$ = magn. flux enclosed by loop $\{① - ②\}$

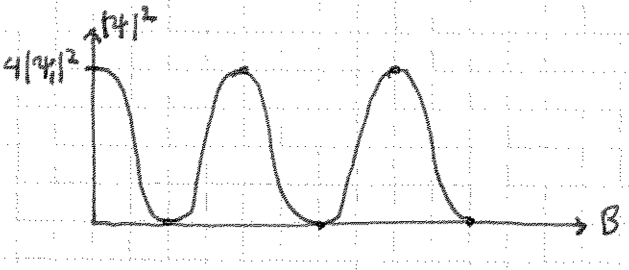
$\Rightarrow$ Total electron wave at detector:

$$\psi = \psi_1 + \psi_2 = \psi_1 + \psi_1 e^{-i\Delta\varphi} = \psi_1\left(1 + e^{-\frac{ie}{\hbar}\Phi}\right)$$

$$|\psi|^2 = |\psi_1|^2 \cdot |1 + e^{-ie\Phi/\hbar}|^2 = 2|\psi_1|^2\{1 + \cos(2\pi\Phi/\Phi_0)\}$$

with $\Phi_0 = h/e \approx 4\cdot 10^{-15}$ Wb "flux quantum"

$\Rightarrow$ at a given position $z$, $|\psi|^2$ is periodic in the B-field of the solenoid:




"Aharonov-Bohm effect"

Note: $B=0$ along electron path $\Rightarrow$ no Lorentz force!

$\Rightarrow$ AB effect is direct QM influence of vector potential $\vec{A}$

What about gauge invariance? OK, since $|\psi|^2$ depends on $\nabla\times\vec{A} = \vec{B}$ ("non local" effect of $\vec{B}$)

Experiments:

Chambers, PRL 5, 3 (1960) : 2-slit exp.

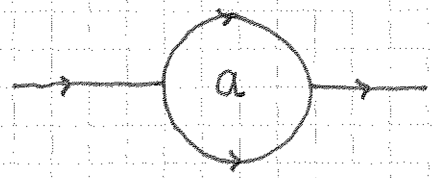
Tonomura et al, PRL 48, 1443 (1982) : electron holography

Webb et al, PRL 54, 2696 (1985) : gold rings

Timp et al, PRL 58, 2814 (1987) : GaAs/AlGaAs

Qualitative discussion of oscillations in conductance G in Webb et al :

- slow aperiodic osc. in $G(B)$ due to interference between paths in the same arm of the ring
- fast periodic osc. in $G(B)$ due to interf. between paths in the two arms of the ring




$\Delta\Phi = h/e$ ; $\Delta\Phi = a \cdot \Delta B$ ($a$ = enclosed area)

$\Rightarrow \Delta B = \dfrac{h/e}{a}$

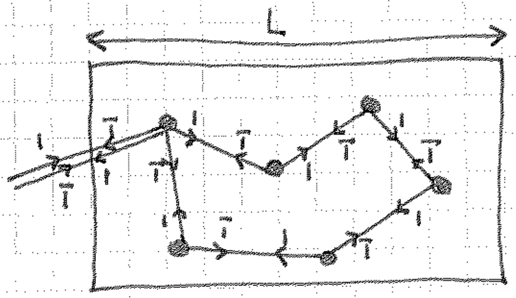
Note: $\vec{B}$ is not zero along electron paths in this experiment (but still AB effect!)

## Weak localization

Assume diffusive but phase coherent transport:

$L \gg \ell_e$ = elastic mean free path

$L \ll \ell_\varphi$ = phase coherence length




● = impurity (⇒ elastic scattering)

1 = path resulting in precise back-scattering
$\bar{1}$ = time reversed version of path 1
} with $\vec{B} = 0$

Equal QM amplitude for 1 and $\bar{1}$: $A_1 = A_{\bar{1}}$

Assume phase coherent paths, $\ell(1) = \ell(\bar{1}) \ll \ell_\varphi$

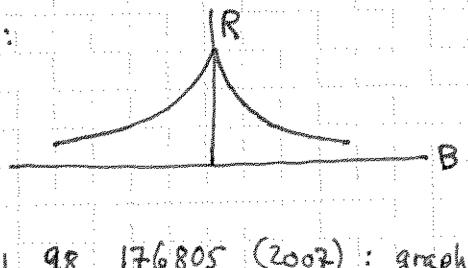
⇒ Prob. of precise back-scattering:

$$P = |A_1 + A_{\bar{1}}|^2 = |2A_1|^2 = 4|A_1|^2 = 4P_1 \,; \quad P_1 = \text{prob. of path } 1$$

(If $\ell(1) > \ell_\varphi$, phase coh. is lost, and $P = P_1 + P_{\bar{1}} = 2P_1$)

⇒ Self-interference gives increased prob. of back-scatt. when transport is phase coherent.

If we turn on $\vec{B} \neq 0$, interf. betw. 1 and $\bar{1}$ is destroyed, and resistance R is reduced! We expect:



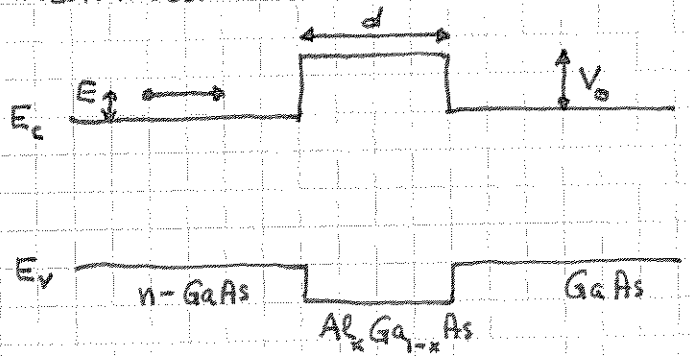

Effect destroyed by incr. temp. T: Shorter $\ell_\varphi$ ⇒ W.L. suppressed.


Refs: Gorbachev et al, PRL 98, 176805 (2007): graphene
Bergmann et al, PRB 28, 2914 (1983): Mg film
Senz et al, PRB 61, R5082 (2000): Si/SiGe quantum well

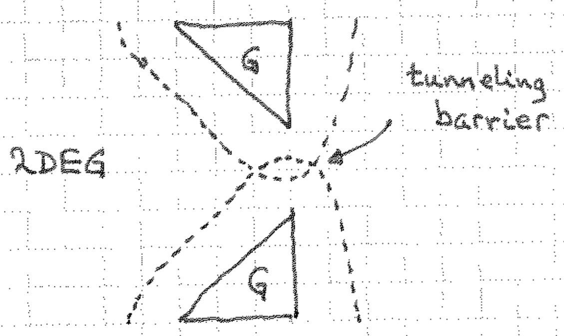
# Resonant tunneling

Exp. realizations of QM tunneling:

- Semiconductor heterostructures



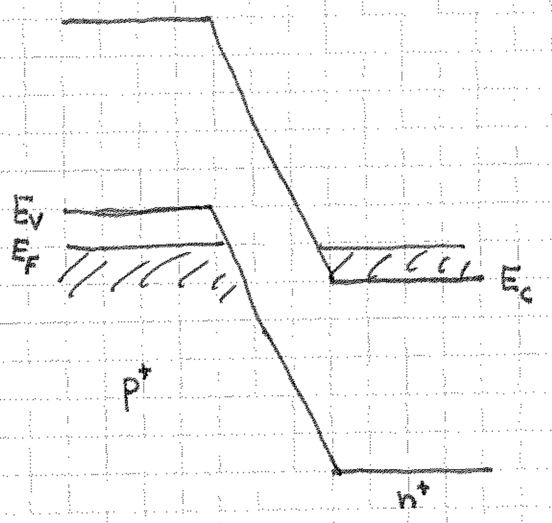

- Quantum point contacts, "pinch-off"




G = gate electrode

------- = limit of classically allowed region

- Tunnel diode, $p^+ - n^+$ junction




(Exp.)
L. Esaki, Phys Rev 109, 603 (1958)

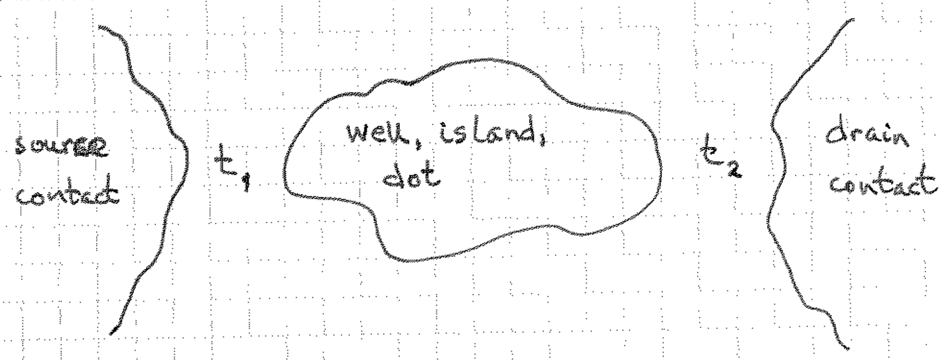
Nobel prize 1973, shared with

I. Giaever (Exp.) and B. D. Josephson (Theory)

"for discoveries regarding tunneling phenomena..."

[I. Giaever: Born in Bergen 1929. Degree in mechanical engineering, NTH, 1952.]

General configuration for resonant tunneling:

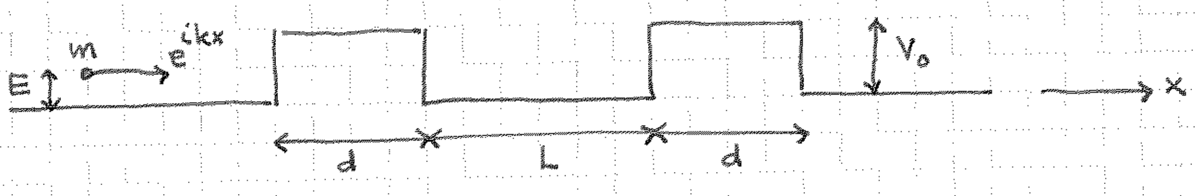



$t_1$ = transmission amplitude for source → well tunneling

$t_2$ = ——— " ——— well → drain —"—

If $|t_{1,2}|^2 \ll 1$, quantum well has "quasi eigenstates" weakly coupled to the source and drain contacts.

Example: Symmetric double barrier semicond. heterostructure [Ex. 11]




$V_0 \sim 200-300$ meV in GaAs / $Al_x\, Ga_{1-x}$ As system with $x \sim 0.3$

$\psi$ and $d\psi/dx$ continuous everywhere

⇒ transmission probability $T(k) = 1/|D(k)|^2$ with

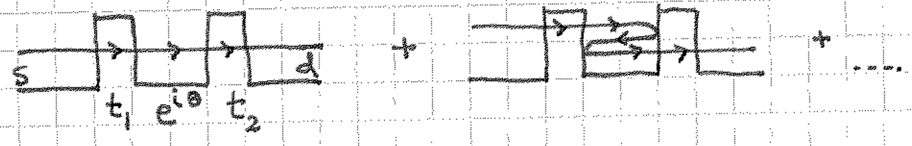
$$D(k) = \cosh^2 \varkappa d + \frac{1}{4}\sinh^2 \varkappa d \left[\sigma^2 \cos 2kL - \delta^2\right]$$
$$+\, i \sinh \varkappa d \left[\delta \cosh \varkappa d + \frac{1}{4} \sinh \varkappa d \sin 2kL\right]$$

$$E = \frac{\hbar^2 k^2}{2m}, \quad V_0 = \frac{\hbar^2 k_0^2}{2m}, \quad \varkappa^2 = k_0^2 - k^2, \quad \sigma = \frac{\varkappa}{k} + \frac{k}{\varkappa}, \quad \delta = \frac{\varkappa}{k} - \frac{k}{\varkappa},$$

$$\sigma^2 = \delta^2 + 4 = \frac{k_0^4}{k^2 \varkappa^2}$$

( see E. H. Hauge et al, PRB 36, 4203 (1987) )

With Feynman paths:




$$\Rightarrow t_{sd} = t_1 e^{i\theta} t_2 + t_1 e^{i\theta} r_2 e^{i\theta} r_1 e^{i\theta} t_2 + \dots$$

$$= t_1 t_2 e^{i\theta} \sum_{j=0}^{\infty} \left( r_1 r_2 e^{2i\theta} \right)^j$$

$$= \frac{t_1 t_2 e^{i\theta}}{1 - r_1 r_2 e^{2i\theta}} = \text{transm. amplitude, source} \rightarrow \text{drain}$$

$\theta$ = phase acquired across well $\approx kL$

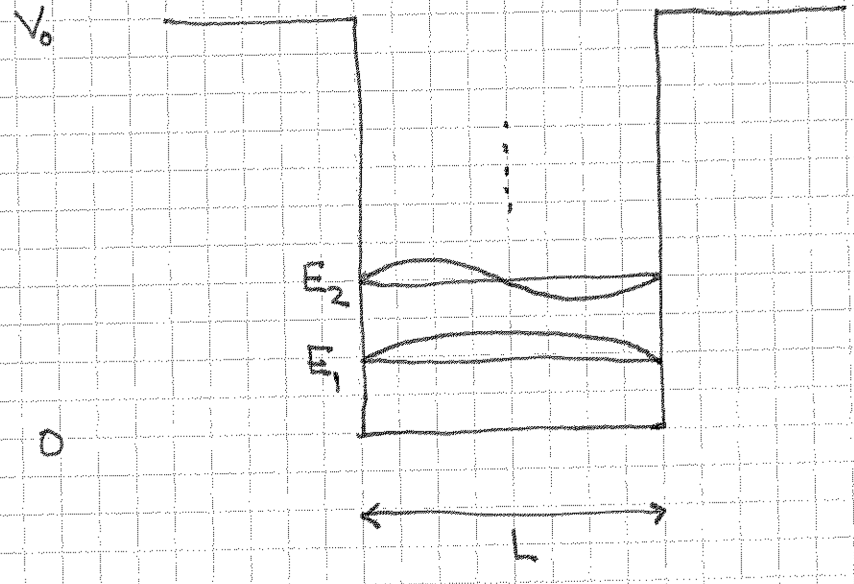
$$\Rightarrow T = |t_{sd}|^2 \approx \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} \cos 2kL} \; ; \quad T_j = |t_j|^2, \quad R_j = 1 - T_j$$

Symmetric $V(x) \Rightarrow T_1 = T_2$, $R_1 = R_2 = 1 - T_1$

$$\Rightarrow T \approx \frac{T_1^2}{2(1-\cos 2kL) - 2T_1(1-\cos 2kL) + T_1^2}$$

If $T_1 \ll 1$: sharp <u>resonances</u>, with $T = 1$ when $\cos 2kL = 1$

$$\Rightarrow 2 \cdot \frac{2\pi}{\lambda_F} \cdot L = 2\pi \cdot n \Rightarrow \lambda_F = \frac{2L}{n}$$



Near resonance, $T(E)$ may be approximated by Lorentzian:

$$T(E) \approx \frac{\Gamma_1 \Gamma_2}{\frac{1}{4}(\Gamma_1+\Gamma_2)^2 + (E-E_j)^2}$$

with

$E_j$ = energy of resonance nr $j$

$\Gamma_i = \hbar \nu T_i \qquad (i=1,2)$

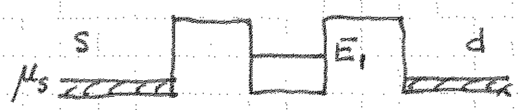
$\nu$ = attempt frequency $= v/2L = (\hbar k/m)/2L$

So far, phase coherent tunneling is assumed: $L \ll l_\varphi$

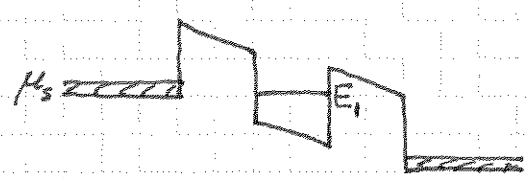
If $L \gtrsim l_\varphi$, we have <u>incoherent</u> "sequential" tunneling, and we must add the <u>probabilities</u> for all possible paths:

$$T_{sd}^{incoh} = T_1 T_2 + T_1 R_2 R_1 T_2 + \dots = \frac{T_1 T_2}{1 - R_1 R_2}$$ (see p. 56)

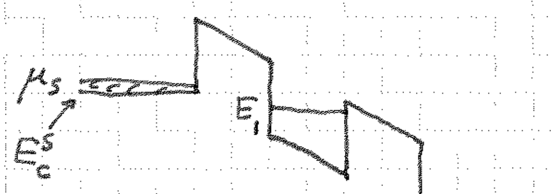
I-V characteristics, qualitatively:




$V_{sd} = 0$, $I = 0$ ①

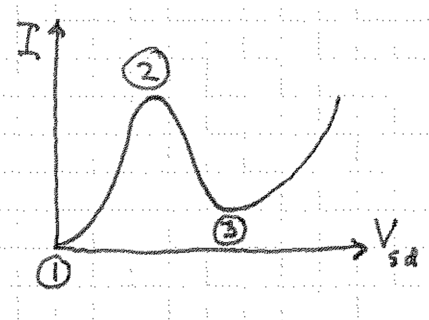



$E_1 \lesssim \mu_s \Rightarrow$ Large $I$ ②




$E_1 < E_c^s \Rightarrow$ small $I$ ③

$\Rightarrow$




Between ② and ③: $dI/dV < 0$, neg. diff. resistance (NDR). Desirable for applications.

End
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