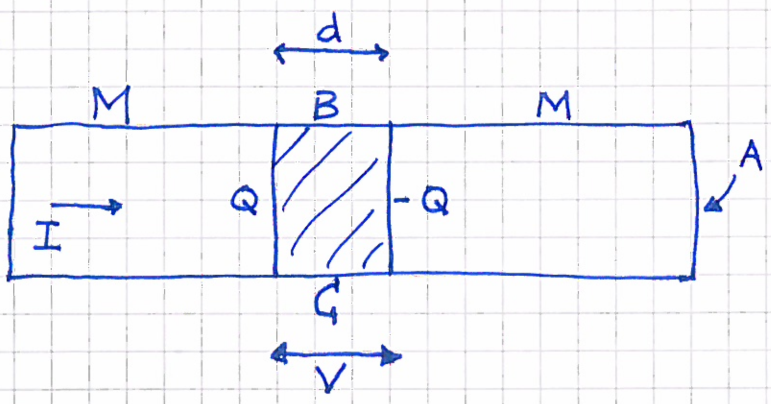
27.03.12

Single Electron Tunneling

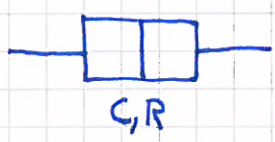
Basics:



$C = \varepsilon A/d$

M : metal, cross-section A

B : barrier ("tunnel junction"), capacitance C and resistance R, circuit symbol:



V: voltage across B

I: current

Q: charge on capacitor

Capacitor charging energy:

$$E_c = \int_0^Q v(q)\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}$$

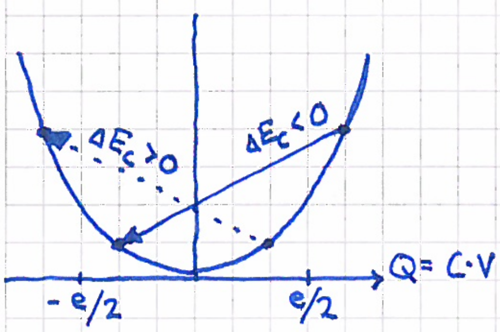
Usually, $k_B T \gg e^2/2C \Rightarrow$ effect of one extra electron on capacitor is negligible (washed out by thermal fluctuations)

If $k_B T \ll e^2/2C$ (made possible with low T and small C), energy change ΔE_c caused by tunneling of a single electron may become a dominant energy in the system.

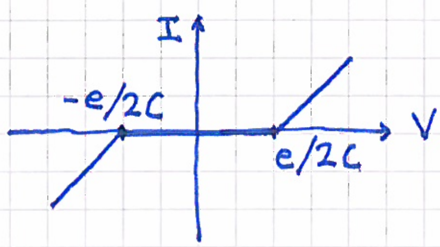
$$\Delta E_c = \frac{(Q \pm e)^2}{2C} - \frac{Q^2}{2C} = \frac{e}{C}\left(\frac{e}{2} \pm Q\right)$$

$\Rightarrow$ tunneling event favored ($\Delta E_c < 0$) if $Q > \frac{e}{2}$

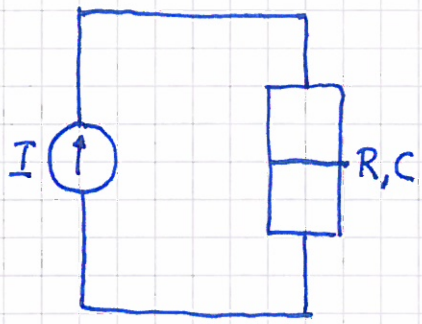
———"——— unfavored ($\Delta E_c > 0$) if $Q < \frac{e}{2}$



$\Rightarrow$ Coulomb blockade!



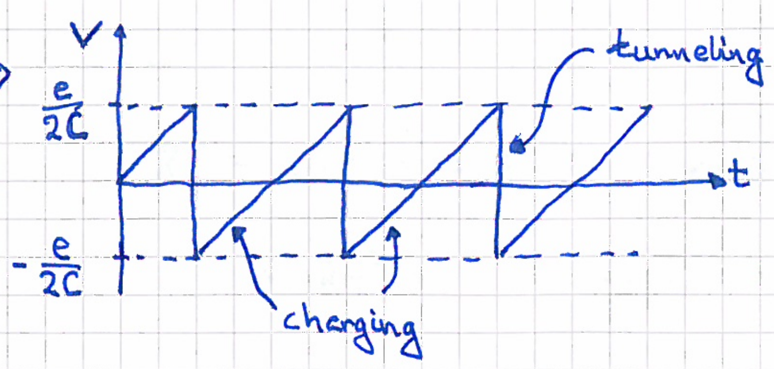
If I is held fixed (by a current source):



$$Q(t) = C \cdot V(t) = \int^t I \, dt' + \Delta Q_{tunnel}$$

$$\Delta Q_{tunnel} = -e$$

$\Rightarrow$

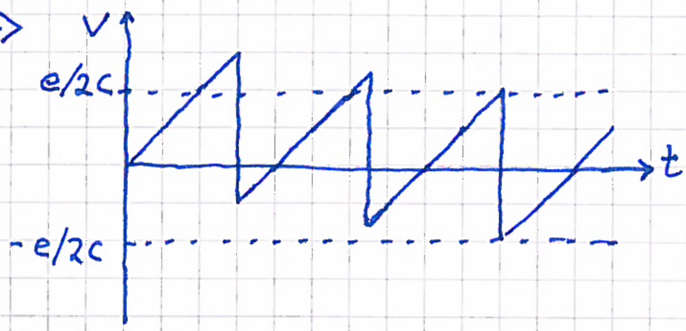


$\Rightarrow$ Single Electron Tunneling oscillations!

$$f_{SET} = I/e$$

Tunneling is a <u>stochastic</u> process

$\Rightarrow$



$\Rightarrow$ peak at $f = I/e$ in the Fourier spectrum of V

Additional requirements for observing Coulomb blockade:

- well-defined tunneling events $\Rightarrow$ must have tunneling time τ smaller than time between two tunneling events $\delta t = e/I$.
  Duration of tunneling $\tau = ?$ Controversial / ill-defined !?
  Probably, $\tau \lesssim 10^{-15}\,s \Rightarrow \delta t \gg \tau$ usually no problem

- negligible quantum fluctuations, i.e., $\delta E \ll e^2/2C$ within time between tunneling events δt.
  Heisenberg: $\delta E \cdot \delta t \geq \hbar/2$
  $\delta t = e/I = e \cdot R/\Delta V = e \cdot R/(e/C) = RC$
  $\Rightarrow RC \geq \frac{\hbar}{2\delta E} \gg \frac{\hbar \cdot 2C}{2e^2} \Rightarrow R \gg \frac{\hbar}{e^2} = \frac{1}{2\pi} \cdot R_Q$

  with $R_Q = h/e^2 \simeq 25.8\,k\Omega$ = resistance quantum

  $\Rightarrow G = R^{-1} \ll 2e^2/h \Rightarrow \sum_\alpha T_\alpha \ll 1$ (Landauer)

  $\Rightarrow$ tunneling probabilities must be <u>small</u>, no problem!

- small thermal fluctuations, $T \ll e^2/2k_B C$

  $\Rightarrow T\,[K] \ll 1/C\,[fF]$, easily obtained
  $f = 10^{-15}$

Early papers:

van Itterbeek and de Greve, Experientia $\underline{3}$, no 7 (1947) (Exp.)

Gorter, Physica $\underline{17}$, 777 (1951) (Theory)

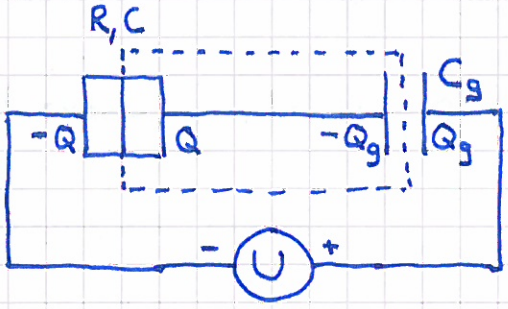
Giaever and Zeller, PRL $\underline{20}$, 1504 (1968) (Exp.)

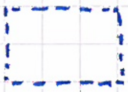
First single electron transistor:

Fulton and Dolan, PRL $\underline{59}$, 109 (1987) (Exp.)

<u>Single Electron Box:</u>

Building block for constructing more complicated devices.



 = electron box, island with quantized net charge

$q = -ne$ ($n = 0, \pm 1, \pm 2, \ldots$ = nr of additional electrons)

 = tunnel junction; $R \gg R_Q$

R, C

 = ideal capacitor ("gate"; $R_g = \infty$)

C_g

End
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