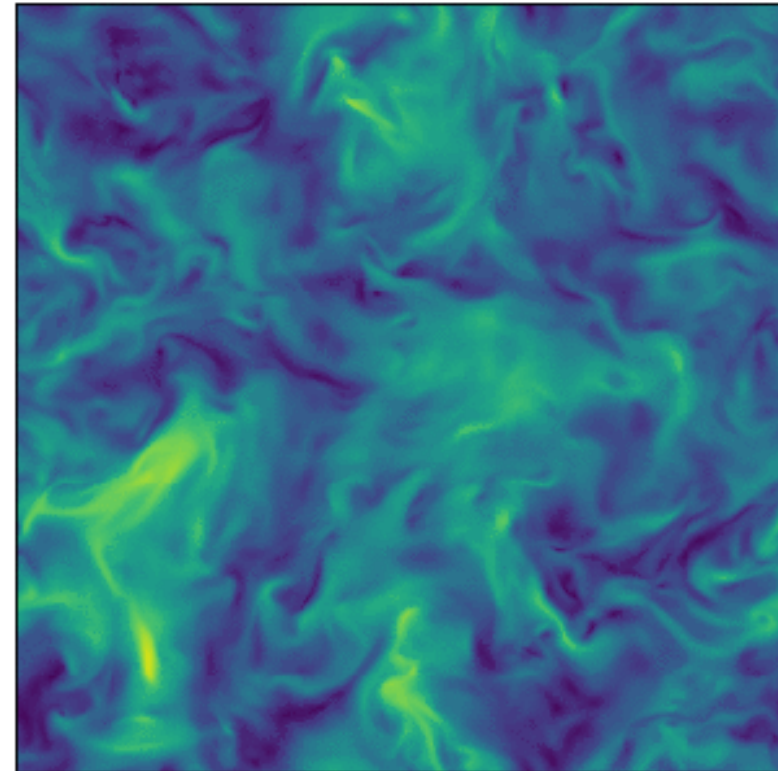


Studies of turbulent diffusion through Direct Numerical Simulation

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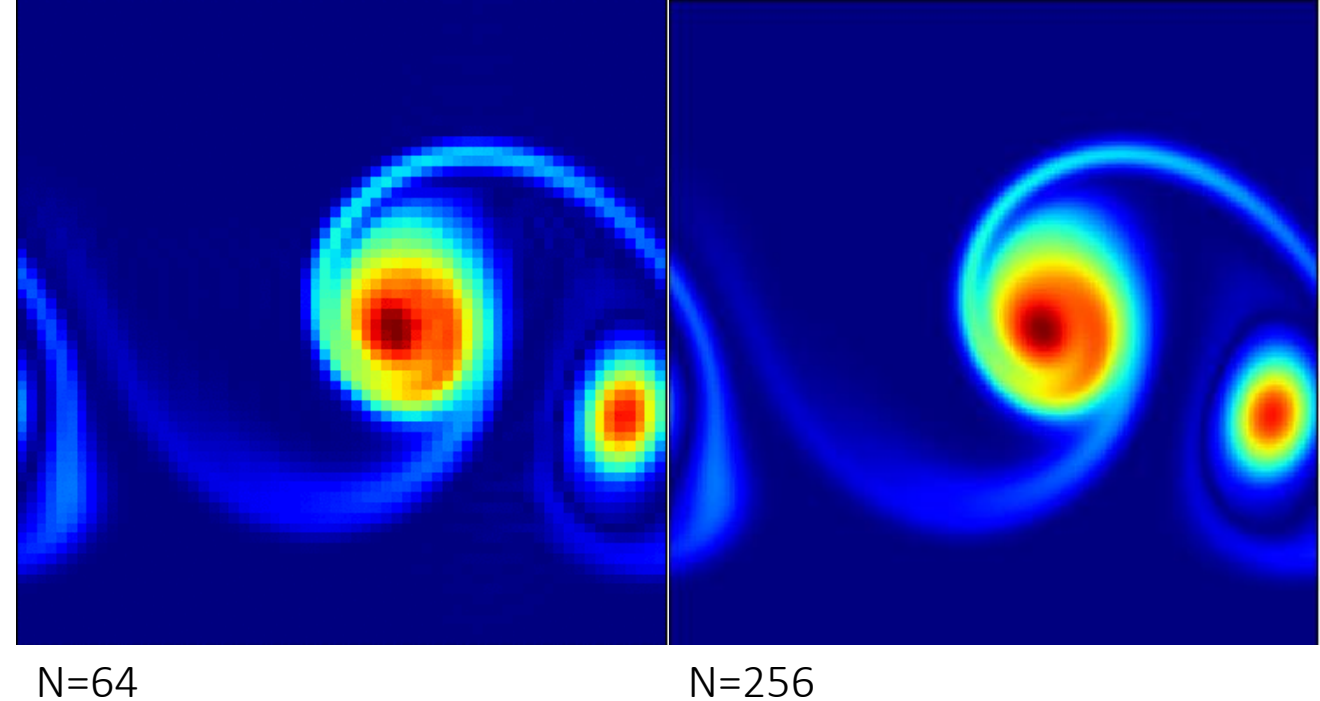


What is turbulence?

- 3D
- Unsteady
- Vorticity
- Diffusive
- Dissipative
- High Reynolds number
- Continuum phenomenon

↓

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} - \mathbf{u} \times \boldsymbol{\omega} \right) = -\nabla(p + \mathbf{u} \cdot \frac{\mathbf{u}}{2}) + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$



Kolmogorov microscale:

$$\eta = l \left(\frac{ul}{\nu} \right)^{\frac{-3}{4}}$$

Spectral method

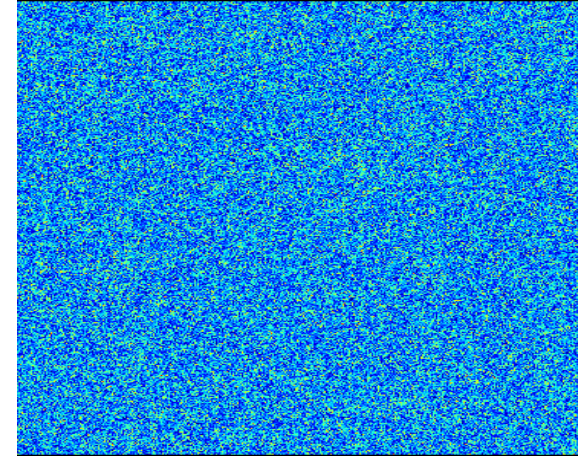
$$\mathbf{u}(\mathbf{x}, t) = \frac{1}{N^3} \sum_{\mathbf{k}} \hat{\mathbf{u}}_{\mathbf{k}}(t) e^{i\mathbf{k} \cdot \mathbf{x}}$$

$$\hat{\mathbf{u}}_{\mathbf{k}}(t) = \sum_{\mathbf{x}} \mathbf{u}(\mathbf{x}, t) e^{i\mathbf{k} \cdot \mathbf{x}}$$

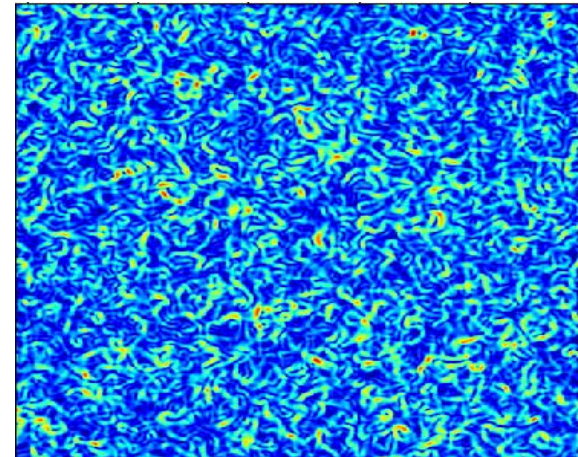
$$\frac{d\hat{\mathbf{u}}_{\mathbf{k}}}{dt} = (\widehat{\mathbf{u} \times \boldsymbol{\omega}})_{\mathbf{k}} - \nu |\mathbf{k}|^2 \hat{\mathbf{u}}_{\mathbf{k}} - \mathbf{k} \frac{\mathbf{k} \cdot (\widehat{\mathbf{u} \times \boldsymbol{\omega}})_{\mathbf{k}}}{|\mathbf{k}|^2}$$

Temporal term solved using your favorite ODE solver!

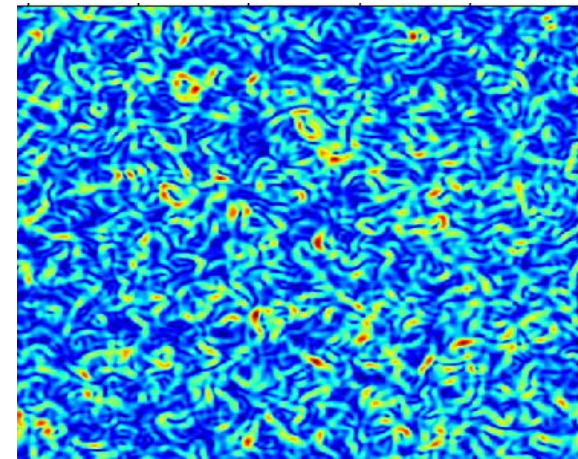
Exponential convergence: $error \propto \left(\frac{L}{N}\right)^N$



N=512
 $t_0 = 0$



N=512
 $t_1 > t_0$



N=512
 $t_2 > t_1$

Summary

- Turbulence is hard
- DNS - all spatial scales resolved
- Spectral methods used for spatial discretization
- Use the turbulent flow field as input in an advection-diffusion equation to simulate transportation of some species.

