Løsning Øving 12

Løsning oppgave 1

$$
\begin{aligned}
y_3 = y_1 + y_2 &= A\cos(kx-\omega t+\varphi_1) + A\cos(kx-\omega t+\varphi_2) \\
&\underset{(K7)\text{ og }(K9)}{=} \text{Re}\left\{Ae^{i(kx-\omega t+\varphi_1)} + Ae^{i(kx-\omega t+\varphi_2)}\right\} \\
&= \text{Re}\left\{Ae^{i(kx-\omega t+(\varphi_1+\varphi_2)/2)}\overbrace{\left(e^{i(\varphi_1-\varphi_2)/2} + e^{-i(\varphi_1-\varphi_2)/2}\right)}^{=2\cos\frac{\varphi_1-\varphi_2}{2}}\right\}
\end{aligned}
$$

(Merk at her er brukt tilsvarende triks som i lign. (4.64) i forelesningene.)

$$
\begin{aligned}
&= \text{Re}\left\{2A\cos\frac{\varphi_1-\varphi_2}{2}e^{i(kx-\omega t+(\varphi_1+\varphi_2)/2)}\right\} \\
&\underset{(K7)}{=} 2A\cos\frac{\varphi_1-\varphi_2}{2}\cos(kx-\omega t+(\varphi_1+\varphi_2)/2) \\
&= \underline{\underline{A_3\cos(kx-\omega t+\varphi_3)}} \qquad (1)
\end{aligned}
$$

der vi har satt:

$$A_3 = \underline{\underline{2A\cos\frac{\varphi_1-\varphi_2}{2}}} \qquad (2)$$

$$\varphi_3 = \underline{\underline{\frac{\varphi_1+\varphi_2}{2}}} \qquad (3)$$

b)

$$
\begin{aligned}
y_3 &= A_1\cos(kx-\omega t+\varphi_1) + A_2\cos(kx-\omega t+\varphi_2) \\
&= \text{Re}\left\{\left(A_1e^{i\varphi_1} + A_2e^{i\varphi_2}\right)e^{i(kx-\omega t)}\right\} \\
&= \text{Re}\left\{A_3e^{i\varphi_3}e^{i(kx-\omega t)}\right\} \\
&= \underline{\underline{A_3\cos(kx-\omega t+\varphi_3)}} \qquad (4)
\end{aligned}
$$

der A_3 og φ_3 er gitt ved at:

$$A_3e^{i\varphi_3} \equiv A_1e^{i\varphi_1} + A_2e^{i\varphi_2} \qquad (5)$$

som gir:
For realdel:

$$A_3\cos\varphi_3 = A_1\cos\varphi_1 + A_2\cos\varphi_2 \qquad (6)$$

For imagniærdel:

$$A_3 \sin\varphi_3 = A_1 \sin\varphi_1 + A_2 \sin\varphi_2 \tag{7}$$

(7) dividert med (6):

$$\tan\varphi_3 = \frac{A_1 \sin\varphi_1 + A_2 \sin\varphi_2}{A_1 \cos\varphi_1 + A_2 \cos\varphi_2} \tag{8}$$

og

$$\varphi_3 = \underline{\underline{\arctan \frac{A_1 \sin\varphi_1 + A_2 \sin\varphi_2}{A_1 \cos\varphi_1 + A_2 \cos\varphi_2}}} \tag{9}$$

A_3 finnes f.eks. ved $A_3 = (A_3^2)^{1/2}$ og

$$\begin{aligned} A_3^2 &= A_3 e^{i\varphi_3} \cdot A_3 e^{-i\varphi_3} \\ &\underset{(5)}{=} \left(A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2}\right) \cdot \left(A_1 e^{-i\varphi_1} + A_2 e^{-i\varphi_2}\right) \\ &= A_1^2 + A_2^2 + A_1 A_2 e^{i(\varphi_1 - \varphi_2)} + A_1 A_2 e^{-i(\varphi_1 - \varphi_2)} \\ &= \underline{\underline{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\varphi_1 - \varphi_2)}} \end{aligned} \tag{10}$$

c) På helt tilsvarende vis som i pkt. b:

$$\begin{aligned} y_N &= \sum_i A_i \cos(kx - \omega t + \varphi_i) \\ &= \mathrm{Re}\left\{ \left(\sum_i A_i e^{i\varphi_i} \right) \cdot e^{i(kx - \omega t)} \right\} \\ &= \mathrm{Re}\left\{ A_N e^{i\varphi_N} e^{i(kx - \omega t)} \right\} \\ &= \underline{\underline{A_N \cos(kx - \omega t + \varphi_N)}} \end{aligned} \tag{11}$$

der A_N og φ_3 er gitt ved at:

$$A_N e^{i\varphi_N} \equiv \sum_i A_i e^{i\varphi_i} \tag{12}$$

som helt tilsvarende som i pkt. b gir:

$$\varphi_3 = \underline{\underline{\arctan\left(\frac{\sum_i A_i \sin\varphi_i}{\sum_i A_i \cos\varphi_i} \right)}} \tag{13}$$

og

$$A_N^2 = \left(\sum_{i=1}^{N} A_i e^{i\varphi_i} \right) \cdot \left(\sum_{j=1}^{N} A_j e^{-i\varphi_j} \right) = \underline{\underline{\sum_{i=1}^{N} \sum_{j=1}^{N} A_i A_j e^{i(\varphi_i - \varphi_j)}}} \tag{14}$$

Løsning oppgave 2

a) Posisjonsvektor for punktet = $\mathbf{r}_p$

$$\omega t - \mathbf{k}\cdot\mathbf{r}_p + \phi = \text{konstant}$$
$$\omega\, dt - \mathbf{k}\cdot d\mathbf{r}_p = 0$$
$$\mathbf{k}\cdot\frac{d\mathbf{r}_p}{dt} = k\cdot\frac{dr_p}{dt} = \omega$$
$$\underline{\underline{\frac{dr_p}{dt} = \frac{\omega}{k}}} \quad \text{q.e.d.}$$

b) Kompleks representasjon (fysikalsk størrelse er her imaginærdelen):

$$E_R = E_0 e^{j\omega t}\left(e^{j\phi} + e^{j2\phi} + e^{j3\phi} + \cdots + e^{jN\phi}\right)$$

Parentesen er summen av en geometrisk rekke.

$$E_R = E_0 e^{j\omega t} e^{j\phi}\frac{1-e^{jN\phi}}{1-e^{j\phi}}$$

Minste verdi for ϕ som gir $E_R = 0$ blir: $\underline{\underline{\phi = 2\pi/N}}$
(For $\phi = 0$ blir $E_R = NE_0\sin\omega t$)

c) Akustisk intensitet:

$$[I_a] = \left[\frac{p^2}{\rho c}\right] = \frac{\frac{\mathrm{N}^2}{\mathrm{m}^4}}{\frac{\mathrm{kg}}{\mathrm{m}^3}\frac{\mathrm{m}}{\mathrm{s}}} = \frac{\mathrm{N}^2}{\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}}} = \frac{\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\mathrm{N}}{\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}}} = \underline{\underline{\frac{\mathrm{W}}{\mathrm{m}^2}}}$$

Elektromagnetisk intensitet:

$$[I_{\mathrm{em}}] = [\varepsilon_0 c E^2] = \frac{\mathrm{F}}{\mathrm{m}}\cdot\frac{\mathrm{m}}{\mathrm{s}}\cdot\left(\frac{\mathrm{V}}{\mathrm{m}}\right)^2 = \frac{\mathrm{CV}^2}{Vsm} = \frac{\mathrm{AV}}{\mathrm{m}^2} = \frac{\mathrm{W}}{\mathrm{m}^2}$$

Altså:

$$\underline{\underline{[I_{\mathrm{em}}] = [I_a] = \frac{\mathrm{W}}{\mathrm{m}^2}}} \qquad \text{q.e.d.}$$

Løsning oppgave 3

Vi har fra lign. (4.16), (4.17), (4.18) og (4.13) i forelesningene:

$$\begin{aligned}
E_\theta &= E_1 + E_2 = E_0[\cos(kr - \omega t - \varphi') + \cos(k(r + \Delta r) - \omega t - \varphi')] \\
&= \mathrm{Re}\left\{E_0\left[e^{i(kr-\omega t-\varphi')} + e^{i(k(r+\Delta r)-\omega t-\varphi')}\right]\right\} \\
&= \mathrm{Re}\left\{E_0 e^{i(kr-\omega t-\varphi'+k\Delta r/2)} \cdot \underbrace{\left(e^{-ik\Delta r/2} + e^{ik\Delta r/2}\right)}_{=2\cos\left(\frac{k\Delta r}{2}\right)}\right\} \\
&= \mathrm{Re}\left\{2E_0\cos\left(\frac{k\Delta r}{2}\right) e^{i(kr-\omega t-\varphi'+k\Delta r/2)}\right\} \\
&= 2E_0\cos\left(\frac{k\Delta r}{2}\right)\cos(kr - \omega t - \varphi' + k\Delta r/2) \\
&= \underline{\underline{E_{\theta_0}\cos(kr - \omega t - \varphi' + \frac{k\Delta r}{2})}}
\end{aligned}$$

som er det samme som lign. (4.21) og (4.22) i forelsning.

Løsning oppgave 4

Ligning (4.74) i forelesningene gir:

$$\frac{D}{A} \approx \Delta\theta = 1.22\frac{\lambda}{d} = 1.22\frac{\lambda_0/n}{d} \quad \Rightarrow$$

$$A = \frac{nDd}{1.22\lambda_0} = \frac{1.33 \cdot 1.4 \cdot 0.006}{1.22 \cdot 580 \cdot 10^{-9}}\mathrm{m} = \underline{\underline{15.8\ \mathrm{km}}}$$