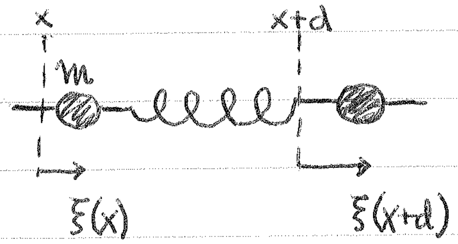
(FGT 14.5, AF 28.10, YF 15.5)

(LL 10.5, TM 15.2

Energi transportert med bølge



Kinetisk energi til m i $E_k = \frac{1}{2} m \left(\frac{\partial \xi}{\partial t}\right)^2$

Potensiell energi i fjær til høyre for m i $E_p = \frac{1}{2} k \left[\xi(x+d) - \xi(x)\right]^2$

$\xi(x+d) \approx \xi(x) + d \cdot \frac{\partial \xi}{\partial x} \quad \Rightarrow E_p \approx \frac{1}{2} k d^2 \left(\frac{\partial \xi}{\partial x}\right)^2$

$\xi(x,t) = \xi(x - vt)$ for bølge i pos. x-retn.

$$\left.\begin{aligned} \Rightarrow \frac{\partial \xi}{\partial t} &= \frac{\partial \xi}{\partial (x-vt)} \cdot \underbrace{\frac{\partial (x-vt)}{\partial t}}_{-v} = -v \frac{\partial \xi}{\partial (x-vt)} \\ \frac{\partial \xi}{\partial x} &= \frac{\partial \xi}{\partial (x-vt)} \underbrace{\frac{\partial (x-vt)}{\partial x}}_{1} = \frac{\partial \xi}{\partial (x-vt)} \end{aligned}\right\} \Rightarrow \frac{\partial \xi}{\partial t} = -v \frac{\partial \xi}{\partial x}$$

La oss sette $X = x - vt$. Ettersom ξ kun avhenger av X, har vi

$$\frac{\partial \xi}{\partial X} = \frac{d\xi}{dX}$$

og dermed $\frac{\partial \xi}{\partial t} = -v \frac{d\xi}{dX}$, $\frac{\partial \xi}{\partial x} = \frac{d\xi}{dX}$

$$\left.\begin{aligned} \Rightarrow E_k &= \frac{1}{2} m \left(\frac{d\xi}{dX}\right)^2 v^2 = \frac{1}{2} m \cdot \frac{kd^2}{m} \left(\frac{d\xi}{dX}\right)^2 = \frac{1}{2} k d^2 \left(\frac{d\xi}{dX}\right)^2 \\ E_p &= \frac{1}{2} k d^2 \left(\frac{d\xi}{dX}\right)^2 \end{aligned}\right\} \Rightarrow E_k = E_p$$

Hit 16.09.06