11.10.06

4. Bølgeligning for $\vec{E}$ og $\vec{B}$ i vakuum

Vakuum : $\rho = 0$, $\vec{j} = 0$

$$\Rightarrow \quad \nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Matematisk identitet: $\nabla \times \nabla \times \vec{G} = \nabla(\nabla \cdot \vec{G}) - \nabla^2 \vec{G}$

der

$$\nabla(\nabla \cdot \vec{G}) = \frac{\partial}{\partial x}(\nabla \cdot \vec{G})\hat{x} + \frac{\partial}{\partial y}(\nabla \cdot \vec{G})\hat{y} + \frac{\partial}{\partial z}(\nabla \cdot \vec{G})\hat{z}$$

og

NB!

$$\nabla^2 \vec{G} = \frac{\partial^2 \vec{G}}{\partial x^2} + \frac{\partial^2 \vec{G}}{\partial y^2} + \frac{\partial^2 \vec{G}}{\partial z^2}$$

Siden $\nabla \cdot \vec{E} = \nabla \cdot \vec{B} = 0$ i vakuum:

$$\nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} \quad , \qquad \nabla \times \nabla \times \vec{B} = -\nabla^2 \vec{B}$$

Faraday || Henry

$$-\nabla \times \frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) \overset{\text{Ampere}}{\underset{\text{Maxwell}}{=}} -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Ampere || Maxwell

$$\mu_0 \varepsilon_0 \nabla \times \frac{\partial \vec{E}}{\partial t} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E}) \overset{\text{Faraday}}{\underset{\text{Henry}}{=}} -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\Rightarrow \quad \boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \qquad \boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Som er bølgeligninger for $\vec{E}$ og $\vec{B}$ (i tre romlige dimensjoner).

Bølgehastighet: $v = 1/\sqrt{\mu_0 \varepsilon_0} \approx 3 \cdot 10^8$ m/s $= c =$ lyshastigheten i vakuum