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Kan bestemme b_n ved å integrere over hvilken som helst periode $\Rightarrow$ velger fra $-\pi$ til π :

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\varphi)\sin n\varphi \, d\varphi = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\varphi}{\pi} \sin n\varphi \, d\varphi$$

Delvis integrasjon: $u = \varphi \qquad v' = \sin n\varphi$
$u' = 1 \qquad v = -\cos(n\varphi)/n$

$$\Rightarrow b_n = \frac{1}{\pi^2}\left\{ \Big|_{-\pi}^{\pi} -\frac{\varphi\cos n\varphi}{n} + \underbrace{\frac{1}{n}\int_{-\pi}^{\pi}\cos n\varphi \, d\varphi}_{=0} \right\}$$

$$= \frac{1}{\pi^2}\left\{ -\frac{\pi}{n}\underbrace{\cos n\pi}_{(-1)^n} + \frac{(-\pi)}{n}\underbrace{\cos(-n\pi)}_{(-1)^n} \right\} = \frac{2}{\pi n}(-1)^{n+1} \qquad (n=1,2,\ldots)$$

$$\Rightarrow f(\varphi) = \sum_{n=1}^{\infty} \frac{2}{\pi n}(-1)^{n+1}\sin n\varphi$$

$$= \frac{2}{\pi}\left\{ \sin\varphi - \frac{1}{2}\sin 2\varphi + \frac{1}{3}\sin 3\varphi - \frac{1}{4}\sin 4\varphi \ldots \right\}$$

Med $\varphi = \omega_0 t = \frac{2\pi}{T} t$:

Laveste frekvens : $\omega_1 = \frac{2\pi}{T} = \omega_0$; Fourier-amplitude (-koeffisienter) : $\frac{2}{\pi}$

2. —"— : $\omega_2 = 2\omega_0$; ——— " ——— : $-\frac{1}{\pi}$

3. —"— : $\omega_3 = 3\omega_0$; ——— " ——— : $\frac{2}{3\pi}$

⋮

n. —"— : $\omega_n = n\omega_0$; ——— " ——— : $\frac{2}{n\pi}(-1)^{n+1}$