

Fourier-integral:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$= \frac{1}{T} \int_0^T f(u) du + \sum_{n=1}^{\infty} \left[\frac{2}{T} \int_0^T f(u) \cos(n\omega_0 u) du \right] \cos(n\omega_0 t)$$

$$+ \sum_{n=1}^{\infty} \left[\frac{2}{T} \int_0^T f(u) \sin(n\omega_0 u) du \right] \sin(n\omega_0 t)$$

$$\omega_n = n\omega_0$$

$$\Delta\omega = \omega_n - \omega_{n-1} = \omega_0 = \frac{2\pi}{T} \Rightarrow \frac{2}{T} = \frac{1}{\pi} \Delta\omega$$

$$\sum_{n=1}^{\infty} \Delta\omega \cdot y(\omega_n) \xrightarrow[\substack{\Delta\omega \rightarrow 0 \\ (T \rightarrow \infty)}]{} \int_0^{\infty} d\omega y(\omega)$$

$$T \rightarrow \infty$$

$$\Rightarrow f(t) = \frac{1}{\pi} \int_0^{\infty} \cos \omega t \int_0^{\infty} f(u) \cos \omega u du dw$$

$$+ \frac{1}{\pi} \int_0^{\infty} \sin \omega t \int_0^{\infty} f(u) \sin \omega u du dw$$

$$[\text{antar } a_0 = \frac{1}{T} \int_0^T f(u) du \rightarrow 0, \text{ dvs antar } \int_0^{\infty} f(u) du < \infty]$$

Dvs: Vilkårlig, ikke-periodisk, $f(t)$ inneholder generelt "alle" frekvenser.

Fourier-spektret er kontinuerlig, med Fourier-koeffisienter

$$a(\omega) = \int_0^{\infty} f(u) \cos \omega u du; \quad b(\omega) = \int_0^{\infty} f(u) \sin \omega u du$$

$$f(t) = \frac{1}{\pi} \int_0^{\infty} a(\omega) \cos \omega t dw + \frac{1}{\pi} \int_0^{\infty} b(\omega) \sin \omega t dw$$