Løsning Øving 6

Løsning oppgave 1

Fra lign. (1.94) i forelesningene har vi

$$\beta \text{ (i dB)} = 10\log(I/I_0). \tag{1}$$

For to lydintensiteter I_1 og I_2 som adskiller seg med $\Delta\beta$ dB må vi da ha:

$$\Delta\beta = 10\log\frac{I_2}{I_0} - 10\log\frac{I_1}{I_0} = 10(\log I_2 - \log I_0 - \log I_1 + \log I_0) = 10\log\frac{I_2}{I_1}$$

som gir

$$\frac{I_2}{I_1} = 10^{\frac{\Delta\beta}{10}}. \tag{2}$$

Fra lign. (1.93) i forelesningene:

$$I \propto p_A^2$$

som gir

$$\frac{p_{A_2}}{p_{A_1}} = \left(\frac{I_2}{I_1}\right)^{1/2} = 10^{\frac{\Delta\beta}{20}}. \tag{3}$$

a) For $\Delta\beta = 10$ dB gir henholdsvis (2) og (3)

$$\frac{I_2}{I_1} = 10^{10/10} = \underline{\underline{10}}$$
$$\frac{p_{A_2}}{p_{A_1}} = 10^{1/2} \approx \underline{\underline{3.16}}$$

b) For $\Delta\beta = 20$ dB:

$$\frac{I_2}{I_1} = 10^{20/10} = \underline{\underline{100}}$$
$$\frac{p_{A_2}}{p_{A_1}} = 10^{10/10} = \underline{\underline{10}}$$

Løsning oppgave 2

a)

$$\lambda = \frac{v}{\nu} = \frac{331}{1000}\text{m} = \underline{\underline{0.331 \text{ m}}}$$

b) Vi har fra forelesningene lign. (1.93):

$$I = \frac{1}{2}\rho v \omega^2 D_0^2 = \frac{1}{2}\frac{p_A^2}{\rho v}$$

som gir henholdsvis for utsvingsamplituden og trykkamplituden:

$$D_0 = \left(\frac{2I}{\rho v \omega^2}\right)^{1/2} = \left(\frac{2 \cdot 1.00 \cdot 10^{-6}}{1.293 \cdot 331 \cdot (2\pi \cdot 1000)^2}\right)^{1/2} \text{ m} = 1.088 \cdot 10^{-8} \text{ m} = \underline{\underline{10.8 \text{ nm}}}$$

$$\begin{aligned} p_A &= (2I\rho v)^{1/2} = (2 \cdot 1.00 \cdot 10^{-6} \cdot 1.293 \cdot 331)^{1/2} \text{ N/m}^2 \\ &= 0.0293 \text{ N/m}^2 = 0.0293 \text{ Pa} = \underline{\underline{29.3 \text{ mPa}}} \end{aligned}$$

c) For $x = 0$ og $t = 0$:

$$\begin{aligned} D(x,t) &= D_0 \cos(0) = D_0 = \underline{\underline{10.8 \text{ nm}}} \\ p(x,t) &= p_A \sin(0) = \underline{\underline{0}} \end{aligned}$$

For $x = 0$ og $t = 2.50 \cdot 10^{-4}$ s $= \frac{T}{4}$:

$$\begin{aligned} D(x,t) &= D_0 \cos\left(-2\pi \frac{1}{T} \cdot \frac{T}{4}\right) = D_0 \cos\frac{\pi}{2} = \underline{\underline{0}} \\ p(x,t) &= p_A \sin\left(-\frac{\pi}{2}\right) = -p_A = \underline{\underline{-29.3 \text{ mPa}}} \end{aligned}$$

29.3 mPa er maksimalverdi for $p(x,t)$. Dermed:

$$P_{\max} = p_0 + p_A = (1.013 \cdot 10^5 + 0.0293) \text{ Pa} = 1.013 \cdot 10^5 \text{ Pa}$$

og det er altså svært langt fra at totaltettheten forandres vesentlig pga. lydbølgen.

d)

$$\begin{aligned} \overline{P} &= I \cdot A = (1.00 \cdot 10^{-6} \cdot 100 \cdot 10^{-6})\text{W} = 1.00 \cdot 10^{-10} \text{ W} = \underline{\underline{100 \text{ pW}}} \\ \beta &= 10 \log \frac{I}{I_0} = 10 \log \frac{10^{-6}}{10^{-12}} = \underline{\underline{60 \text{ dB}}} \end{aligned}$$

e) For $I = 1.00 \cdot 10^{-12}$ W/m^2 (ved hjelp av resultatene i pkt. b):

$$D_0 = 10.8 \text{ nm} \cdot \left(\frac{10^{-12}}{10^{-6}}\right)^{1/2} = \underline{\underline{10.8 \text{ pm}}} \quad \text{(dvs. mindre enn radien til atomer)}$$

$$p_A = 29.3 \text{ mPa} \cdot \left(\frac{10^{-12}}{10^{-6}}\right)^{1/2} = \underline{\underline{29.3\ \mu\text{Pa}}}$$

For $I = 1.00$ W/m^2:

$$D_0 = 10.8 \text{ nm} \cdot \left(\frac{1}{10^{-6}}\right)^{1/2} = \underline{\underline{10.8\ \mu\text{m}}}$$

$$p_A = 29.3 \text{ mPa} \cdot \left(\frac{1}{10^{-6}}\right)^{1/2} = \underline{\underline{29.3 \text{ Pa}}}$$

Løsning oppgave 3

$$\overline{P} = 8 \text{ W} \cdot 0.03 = 0.24 \text{ W}$$

$$I = \frac{\overline{P}}{A} = \frac{0.24}{\frac{1}{2} \cdot 4\pi \cdot 10^2} \text{W/m}^2 = \underline{\underline{3.8 \cdot 10^{-4} \text{ W/m}^2}}$$

$$\beta = 10 \log \frac{3.8 \cdot 10^{-4}}{10^{-12}} = \underline{\underline{86 \text{ dB}}}$$