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A new light-intensity-dependent Brewster angle caused by programmed defects

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Abstract

It is known that by using induced electric and magnetic dipole moments as the microscopic sources for the macroscopic optical phenomena, both the Fresnel equations and consequently, the Brewster angle formula, can be rigorously reproduced. Motivated by such physical intuition and mathematical rigor, we demonstrate that novel materials of variant Brewster angles can be created by embedding, within selected uniform isotropic host materials, pre-determined, optically responsive defects at the prescribed orientations in the form of permanent dipoles. Following this, a peculiar incident-light-intensity-dependent Brewster angle (θ_B) is discovered. In addition, it is even possible to render $\theta_B = 0$ via proper manipulation of the permanent polarization strength and its orientation, as well as the magnetic permeability. The feasibility of such active modifying approach using pre-determined defects should be very much enhanced as we are entering the era of nanometer technologies. Thus, we will not be surprised to see the birth of various new materials, novel devices and applications originated from such defect-engineering approach, in the coming days. \bigcirc 2007 Elsevier Ltd. All rights reserved.

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1. Introduction

In optics, it is well-known that when a visible light beam, e.g., traveling from air (or more strictly, vacuum) into a piece of smooth flat glass at an angle relative to the normal of the air-glass interface, some proportion of the light will be bounced off at the reflection angle equal to the incident angle. However, when the light beam is with its oscillating electric field parallel to the plane-of-incidence (POI, i.e., the plane constituted by both propagation vectors of the incident and reflected light waves, as well as the interface normal vector) (called the p-wave), there is a particular incident angle is known as the Brewster angle (θ_B) [1]. In contrast, when the light beam is with its electric field vector perpendicular to the POI (called the s-wave), no such angle

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exists [1]. In fact, this is only true for uniform, isotropic, and nonmagnetic (or equivalently, with its relative magnetic permeability (μ_r) equal to unity) materials such as the above glass piece. Indeed, it is known that for magnetic materials, there may instead exist Brewster angles for the s-waves, while none for the p-waves [1].

Traditionally, the Brewster angle is a fixed property of the material in question with respect to a given light frequency (or color) of interest. Namely, there is a one-toone correspondence between the Brewster angle and the incident light frequency. However, it is the main task of this paper to show that the Brewster angle of each specific material can in principle be modified into a controllable variable, even dynamically, if a post-process microscopic method called the "*dipole engineering*" is applied on that material.

In order to clearly demonstrate the intricacies of the mechanism of the proposed *permanent* dipole engineering subsequently, existing result of Doyle [2] is first gone through in details. That is, the Fresnel equations and Brewster angle formula are to be arrived at intuitively but

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rigorously, by viewing all light-wave-induced dipole moments (including both electric and magnetic dipoles) as the microscopic sources causing the observed macroscopic optical phenomena at an interface, as compared to the traditional academic "Maxwell" approach ignoring the dipole picture. Then, equipped with such—developed intuitive and quantitative physical picture, the readers are then ready to appreciate the way those *optically responsive*, *permanent* dipoles are externally implemented into a selected host matter and their effects. Namely, the Brewster angle of a selected host material may be altered, likely at will, and ultimately new optical materials, devices and applications can be created.

2. Brewster angle and "scattering" form of Fresnel equations

Arising from Maxwell's equations (through assuming linear media and adopting monochromatic plane-waves expansion), Fresnel equations provide almost complete quantitative descriptions about the incident, reflected and transmitted waves at an interface, including information concerning energy distribution and phase variations among them [1]. Two of the Fresnel equations related to the reflection coefficients associated with both the p and s components are listed here [1]:

$$r^{s} = \frac{E_{r}^{s}}{E_{i}^{s}} = \frac{n_{i}\mu_{rt}\cos\theta_{i} - n_{t}\mu_{ri}\cos\theta_{t}}{n_{i}\mu_{rt}\cos\theta_{i} + n_{t}\mu_{ri}\cos\theta_{t}},$$
(1)

$$r^{p} = \frac{E_{r}^{p}}{E_{i}^{p}} = \frac{\mu_{ri}n_{t}\cos\theta_{i} - \mu_{rt}n_{i}\cos\theta_{t}}{\mu_{ri}n_{t}\cos\theta_{i} + \mu_{rt}n_{i}\cos\theta_{t}},$$
(2)

where *E* is the electric field, μ_r is the relative magnetic permeability, $n = (\varepsilon_r \mu_r)^{1/2}$ is the index of refraction $(\varepsilon_r$ being the relative dielectric coefficient), superscripts "*p*" and "*s*" stand for the p-wave and s-wave components, while subscripts "*i*", "*r*" and "*t*" denote incident, reflected and transmitted components, respectively. When the incident angle (θ_i) is equal to a particular value (θ_B) , one of the above reflection coefficients would vanish, then such value of the incident angle is known as the Brewster angle (θ_B) . Note that for the most familiar case in which the light wave is incident from vacuum onto a linear nonmagnetic medium $(\mu_r = 1)$, only the p-wave possesses a Brewster angle, not the s-wave.

In the following, to get ready for our proposed idea while without loss of generality, the medium on the incident side is designated to be vacuum (i.e., $n_i = 1$) for simplicity. In addition, to further facilitate our purpose, the Fresnel equations in the equivalent "scattering" form (due to Doyle) are retyped here [2]:

$$\frac{E_t^p}{E_i^p} = \left[\frac{-\sqrt{\mu_r}}{(\mu_r - 1)\sqrt{\varepsilon_r} + (\varepsilon_r - 1)\sqrt{\mu_r}\cos(\theta_t - \theta_i)} \right] \times \left[\frac{2\cos\theta_i\sin(\theta_t - \theta)}{\sin\theta_t} \right],$$
(3)

$$\frac{E_r^p}{E_i^p} = \left[\frac{(\mu_r - 1)\sqrt{\varepsilon_r} - (\varepsilon_r - 1)\sqrt{\mu_r}\cos(\theta_t + \theta_i)}{(\mu_r - 1)\sqrt{\varepsilon_r} + (\varepsilon_r - 1)\sqrt{\mu_r}\cos(\theta_t - \theta_i)} \right] \times \left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} \right],$$
(4)

$$\frac{E_t^s}{E_i^s} = \left[\frac{-\sqrt{\mu_r}}{(\varepsilon_r - 1)\sqrt{\mu_r} + (\mu_r - 1)\sqrt{\varepsilon_r}\cos(\theta_t - \theta_i)} \right] \times \left[\frac{2\cos\theta_i\sin(\theta_t - \theta_i)}{\sin\theta_t} \right],$$
(5)

$$\frac{E_r^s}{E_i^s} = \left[\frac{(\varepsilon_r - 1)\sqrt{\mu_r} - (\mu_r - 1)\sqrt{\varepsilon_r}\cos(\theta_t + \theta_i)}{(\varepsilon_r - 1)\sqrt{\mu_r} + (\mu_r - 1)\sqrt{\varepsilon_r}\cos(\theta_t - \theta_i)} \right] \\
\times \left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} \right].$$
(6)

Namely, the right-hand side of Eqs. (3)–(6) is in the form of $D \times S$, with D being the first bracketed term, representing single-dipole (electric and magnetic) oscillation; while S being the second term, depicting the collective scattering pattern generated by the whole array of dipoles. While S is nonzero, where D vanishes (Eq. (4) or (6)) is the condition for the Brewster angle to arise either for the p-wave or the s-wave, i.e.,

$$\tan^2 \theta_B^p = \frac{\varepsilon_r(\varepsilon_r - \mu_r)}{\varepsilon_r \mu_r - 1},\tag{7}$$

$$\tan^2 \theta_B^s = \frac{\mu_r(\mu_r - \varepsilon_r)}{\varepsilon_r \mu_r - 1}.$$
(8)

Note that if the medium is characterized by $\mu_r > \varepsilon_r$, only the s-wave would encounter the Brewster angle, and in the situation where $\varepsilon_r > \mu_r$, only the p-wave would. Indeed, in the most familiar case of light going from vacuum into a piece of glass whose $\varepsilon_r > \mu_r = 1$, there is the Brewster angle only for the p-wave.

In the following, an alternative derivation of the "scattering" form Fresnel equations (Eqs. (4)–(6)) will be reproduced from Ref. [2] in somewhat details, which stems from the viewpoint of treating induced microscopic (electric and magnetic) dipoles as the effective sources of macroscopic EM waves at the interface. Then, effective ways to embed the proposed permanent dipoles within a host material will be proposed, which will in principle allow us to achieve variant Brewster angles and thus to create novel materials, devices, and new applications.

3. "Scattering" form Fresnel equations from the dipole source viewpoint

Microscopically, all matters including optical materials are made of atoms or molecules, each of which further consists of a positive-charged nucleus (or nuclei) and some orbiting negative-charged electrons. When subjected to the EM field of an impinging light wave, the positive and negative charges separate to form induced electric dipoles along the light electric field, while some electrons further orbit in ways to form induced magnetic dipole moments along the light magnetic field. Note that in this work, only far-fields generated by these dipoles are considered, and the respective orientations of all field quantities are depicted in Fig. 1.

3.1. The p-wave (E parallel to the plane-of-incidence) case

It is well-known from electromagnetism [3–5] that if denoting the induced electric polarization by \vec{P} , the electric displacement is: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$. Comparing it with $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$ (for linear, isotropic media), the polarization can thus be written as (all in space-time configuration) [3]:

$$\vec{P} = \varepsilon_0 (\varepsilon_r - 1) \vec{E}_t^p.$$
(9)

When calculating the field contribution from the electric dipoles alone, it is assumed that $\mu_r = 1$ and thus $n = (\varepsilon_r)^{1/2}$. Putting this into an equivalent form of the Snell's law [2]:

$$n^{2} - 1 = \frac{\sin(\theta_{i} + \theta_{t})\sin(\theta_{i} - \theta_{t})}{\sin^{2}\theta_{t}}$$
(10)

and subsequently, together with Eq. (9), into the Fresnel equation related to E_t/E_i , we obtain an expression in which the incident electric field is expressed as a "consequence" of the microscopic sources—the induced dipoles:

$$E_{ip}^{p} = \left[-\frac{P}{\varepsilon_{0}}\cos\left(\theta_{t} - \theta_{i}\right)\right] \left[\frac{\sin\theta_{t}}{2\cos\theta_{i}\,\sin\left(\theta_{t} - \theta_{i}\right)}\right],\tag{11}$$

where the subscript "p" stands for contribution from the induced electric dipoles. Incorporating Eq. (11) into Eq. (2), the reflected electric field is also expressed to be due to the electric dipoles sources:

$$E_{rp}^{p} = \left[\frac{P}{\varepsilon_{0}}\cos(\theta_{t} + \theta_{i})\right] \left[\frac{\sin\theta_{t}}{2\cos\theta_{i}\sin(\theta_{t} + \theta_{i})}\right].$$
 (12)

Similarly, there are field contributions from the induced magnetic dipoles, which acted through the magnetization \vec{M} . Using the magnetic field strength $\vec{H} = (\vec{B} / \mu_0) - \vec{M}$ and the $E = (\omega/k)B = vB$ relation associated with general



Fig. 1. Orientation of induced dipoles and their fields.

monochromatic plane waves, the magnetization can be written as [3-5]

$$M = (\mu_r - 1) \sqrt{\frac{\varepsilon_0 \varepsilon_r}{\mu_0 \mu_r}} E_t^p.$$
(13)

In considering the magnetic contribution, $\varepsilon_r = 1$ and thus $n = (\mu_r)^{1/2}$ are adopted in Eq. (10), and then, together with Eq. (13), incorporated into the Fresnel equation related to E_t/E_i to arrive at an expression of the magnetic component of the incident electric field as a "consequence" of the microscopic-induced magnetic dipoles:

$$E_{im}^{p} = \left[-\frac{M\sqrt{\mu_{0}}}{\sqrt{\varepsilon_{0}}} \right] \left[\frac{\sin\theta_{t}}{2\cos\theta_{i}\sin(\theta_{t} - \theta_{i})} \right], \tag{14}$$

where the subscript "*m*" stands for contribution from the induced magnetic dipoles. Putting Eq. (14) into Eq. (2), the magnetic component of the reflected electric field can be expressed as due to the induced magnetic dipoles too:

$$E_{rm}^{p} = \left[-\frac{M\sqrt{\mu_{0}}}{\sqrt{\varepsilon_{0}}} \right] \left[\frac{\sin\theta_{t}}{2\cos\theta_{i}\sin(\theta_{t}+\theta_{i})} \right].$$
 (15)

When a light wave impinges on an interface, it causes the excitation of electric and magnetic dipoles throughout the second medium, which in turn collectively give rise to reflected, transmitted, and formally, incident waves at the interface. As depicted in Fig. 1, as long as far fields are concerned, the induced electric and magnetic dipoles can be viewed as aligned along the incident electric and magnetic fields of interface-relevant waves can be conceived as generated by the co-work of electric and magnetic dipoles. (Of course, for the incident wave, this is only formally true, i.e., the incident fields are the "cause" not the "effect" of dipole oscillations.) We thus write

$$E_{i}^{p} = E_{ip}^{p} + E_{im}^{p}.$$
 (16)

Namely, by adding Eqs. (11) and (14), the incident electric field can be formally expressed as due to induced dipole sources P and M:

$$E_i^p = \left[-\frac{P}{\varepsilon_0} \cos(\theta_t - \theta_i) - \frac{M\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \right] \\ \times \left(\frac{\sin \theta_t}{2\cos \theta_i \sin(\theta_t - \theta_i)} \right).$$
(17)

Putting forms of these sources (i.e., Eqs. (9) and (13) in which the induced P and M are expressed in term of E_t^p) back into Eq. (17), it is found that the obtained transmission coefficient of the p-wave is exactly that of Eq. (3).

Similarly, conceiving the reflected electric field as

$$E_r^p = E_{rp}^p + E_{rm}^p. aga{18}$$

Adding Eqs. (12) and (15), the reflected electric field can also be viewed as due to the induced dipole sources 4

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P and M:

$$E_r^p = \left[\frac{P}{\varepsilon_0}\cos(\theta_t + \theta_i) - \frac{M\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\right] \times \left(\frac{\sin\theta_t}{2\cos\theta_i\sin(\theta_t + \theta_i)}\right).$$
(19)

Putting forms of the dipoles sources (i.e., Eqs. (9) and (13) in which *P* and *M* are expressed in terms of E_t^p) back into Eq. (19), and using the newly obtained E_i^p vs. E_t^p relation (i.e., Eq. (17)), it is found that the obtained reflection coefficient of the p-wave is exactly that of Eq. (4), in "scattering" form.

3.2. The s-wave (E perpendicular to the plane-of-incidence) case

Similarly, as depicted in Fig. 1, we can also perceive the incident and reflected electric fields of the s-wave as resulted from those induced electric and magnetic dipoles:

$$E_i^s = \left(-\frac{P}{\varepsilon_0} - \frac{M\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \cos(\theta_t - \theta_i) \right) \\ \times \left(\frac{\sin \theta_t}{2\cos \theta_i \sin(\theta_t - \theta_i)} \right), \tag{20}$$

$$E_r^s = \left(-\frac{P}{\varepsilon_0} + \frac{M\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \cos(\theta_t + \theta_i) \right) \\ \times \left(\frac{\sin \theta_t}{2\cos \theta_i \sin(\theta_t + \theta_i)} \right)$$
(21)

and that the transmission and reflection coefficients are indeed Eq. (5) and (6), respectively, in the "scattering" form.

4. The proposed *permanent* dipoles engineering

4.1. Observations and inspirations

From the above elaboration, it is now better known that the electric and magnetic dipoles are much more than mere pedagogical tools for picturing dielectrics and magnetics, as indeed proved by Doyle [2]. In fact, treating them as microscopic EM wave sources from the outset, the "scattering" form of Fresnel equations (i.e., Eqs. (3)–(6)), and consequently, the Brewster angle formulas (Eqs. (7) and (8)) can all be reproduced. Then, emerging from such details comes our inspired purposes. Namely, and much more importantly, by acquainting ourselves with the role played by these *induced* dipoles (or, the microscopic scattering sources), it is then intuitively straightforward to learn how new macroscopic optical phenomena, such as the new Brewster angle may be generated if extra permanent dipoles were implemented onto whatever host material in discussion. In other words, now the notions of *P* and *M* are further extended to include the total effects resulting from both the *induced* and *permanent* dipoles.

For instance, in Eq. (17), the $\cos(\theta_t - \theta_i)$ factor multiplying on P (but not on M) is due to the fact that the induced polarization P (along E_t^p) has only a fractional contribution to E_i^p determined by the vector projection as shown in Fig. 2, for the p-wave situation. Now, if an external polarization vector P_0 (as the collective result of many imposed electric dipoles) is introduced within a host material, then, e.g., for the p-wave case, all electric dipoles' contribution to E_i^p (i.e., Eq. (17)) is now $P_{induced} \cos(\theta_i - \theta_i) +$ $P_0 \cos(\theta_0 + \theta_i)$, or $\varepsilon_0(\varepsilon_r - 1)E_t^p \cos(\theta_i - \theta_t) + P_0 \cos(\theta_0 + \theta_i)$ (see Fig. 2). Namely, there is now an additional second term resulting from *externally* imposed dipoles. Note that the incident light-driven response of these externally imposed permanent dipoles is frequency dependent, and therefore, the above P_0 really stands for that amount of polarization associated with the relevant optical frequency of interest. In other words, a "DC" polarization will never enter the above equation.

Thus, if the dipole-engineered total contribution is recast in the traditional form, viz., $\varepsilon_0(\tilde{\varepsilon}_r - 1)E_t^p \cos(\theta_i - \theta_t)$, then it is clear that the modified relative dielectric coefficient is equivalently:

$$\tilde{\varepsilon}_r = \frac{P_0}{\varepsilon_0 E_t^p} \frac{\cos(\theta_i + \theta_0)}{\cos(\theta_i - \theta_t)} + \varepsilon_r,$$
(22)

where θ_0 is the angle between the imposed extra polarization vector and the interface plane (see Fig. 2). Thus, by putting Eq. (22) into the p-wave Brewster angle formula (Eq. (7)), a totally untraditional Brewster angle (θ_B) would then emerge:

$$\tan^2 \theta_B^p = \frac{\tilde{\varepsilon}_r(\tilde{\varepsilon}_r - \mu_r)}{\tilde{\varepsilon}_r \mu_r - 1}.$$
(23)



Fig. 2. The p-wave configuration at the interface and the orientation of embedded permanent electric dipoles.

4.2. Justification of the effectiveness and meaningfulness of implementing optically responsive dipoles

A justification of the effectiveness of the proposed *permanent* dipole engineering is straightforward by noting the following fact. Namely, had the original host material been transformed into a new material by adding in a considerable amount of certain second substance, then P_0 in the above really would have stood for the extra induced dipole effect resulting from this second substance.

However, to this end, an inquiry may naturally arise as to whether the outcome of the proposed dipole-engineering approach would be nothing more than having a material of multi-components from the outset. The answer is clearly no, and there are much more meaningful and practical intentions behind the proposed method. First of all, this is a controllable way to make new materials from known materials without having to largely mess around with typically complicated details of manufacturing processes pertaining to each involved material (if the introduced permanent dipoles are noble enough). Indeed, we have been routinely attempting to create various materials by combining multiple substances, and yet have also been very much limited by problems related to chemical compatibility, phase transition, in addition to many processing and economic considerations. Secondly, permanent dipole engineering would further allow delicate, precise means of manipulating the material properties, such as varying the dipole orientation to render desired optical performance on host materials of choice. Thirdly, all existing techniques known to influence dipoles can be readily applied on the now embedded dipoles to harvest new optical advantages, such as by electrically biasing the dipoles to adjust the magnitude of permanent dipole moment (in terms of P_0) in the frequency range of interest.

Another question may also arise as to whether there be a limit for the value of P_0 below which Eq. (22) and thus the above statements still stand. Although such limit has not yet been determined either theoretically or experimentally, it is reasonable to believe that Eq. (22) is at least true when the imposed effect of *permanent* dipoles is mere perturbation to the original host material.

5. Some preliminary conjectures on possible applications following the proposed method

As implied by Eq. (23), it is possible to render $\theta_B = 0$ by making either $\tilde{\varepsilon}_r = 0$ or $= \mu_r$ through proper manipulations of P_0 and θ_0 , as well as optionally μ_r . In such cases, there will be no reflection at normal incidence. For example, if the host matter is nonmagnetic ($\mu_r = 1$), then we have the null Brewster angle under the condition:

$$P_0(\omega) = \frac{\varepsilon_r(\omega)\varepsilon_0 E_t^p(\omega)}{|\cos \theta_0|} \text{ with } \theta_0 > \frac{\pi}{2}.$$
(24)

In this case, it is somewhat hard to do with $\theta_0 \approx \pi/2$ since the needed *permanent* polarization P_0 would have to be very large, even if the validity of Eqs. (22) and (23) still stand, which is unlikely. On the other hand, making $\theta_0 \approx \pi$ would only require a minimal implementation of *permanent* polarization of $P_0 \approx \varepsilon_r \varepsilon_0 E_t^p$.

An inspection of both Eqs. (22) and (23) further reveals that the new Brewster angle is dependent on the transmitted electric field E_i^p (or reasonably, on the incident light power) in addition to the aforementioned dependence on orientation of the embedded *permanent* dipoles. That is, even for the perturbative case, Eq. (22) still weirdly implies that the deviation from the traditional Brewster angle is significant when the incident light is fairly weak in power. In other words, the new Brewster angle now varies according to the power of the incident light. For example, for a nonmagnetic ($\mu_r = 1$) host matter embedded with a fixed amount of permanent dipoles, and, then an incident light of less power, with $\theta_i + \theta_0 < \pi/2$ (see Eq. (22)), would experience a larger Brewster angle than its counterpart of a larger incident power.

For the s-wave case, the derivations are somewhat similar, only that the results would be different owing to different field configurations. Likewise, similar microscopic engineering can be performed on the magnetic side by externally introducing a magnetization M_0 , as the collective result of many externally imposed magnetic dipoles.

6. Summary and conclusions

The readers were first familiarized with the fact that by employing induced dipole moments as the microscopic sources for the macroscopic optical phenomena, both the Fresnel equations and Brewster angle formula [1] can be rigorously and quantitatively reproduced [2]. From the grasp of this much intuitive physical picture, it was demonstrated that novel materials of variant Brewster angles can be created from converting ordinary host materials through embedding within them permanent dipoles coherently responsive to the optical frequency of interest. Then, several preliminary conjectures were presented concerning the possible applications as a result of the proposed method. The feasibility of this dipole engineering approach should be very much enhanced as we are entering the era of nanometer technologies. Hence, we ought not to be surprised to see the birth of new materials, novel devices and applications originated from such an approach in years down the road.

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