NORGES TEKNISK-
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EXAM TFY4160 BØLGEFYSIKK
Mandag 3. desember 2007 kl. 0900 - 1300
English version

Hjelpemiddel: C

- K. Rottmann: Matematisk formelsamling (alle språk).
- O. Øgrim og B. E. Lian: Størrelser og enheter i fysikk og teknikk, eller C. Angell og B. E. Lian: Fysiske størrelser og enheter.
- Typegodkjend kalkulator, med tomt minne, i henhold til liste utarbeidd av NTNU. (HP30S eller liknende.)

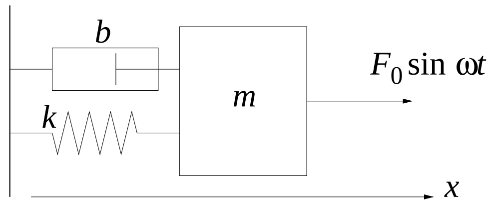
Page 2 - 6: Exercises.
Page 7 - 15: Formulas.

There are 7 questions. The weight of each question is given. Vectors are given in **bold** letters. Unit vectors are given with a hat above the symbol.

The marks are ready in early January.

OPPGAVE 1 [Counts 20%]

A mass m is connected to a spring with spring constant k. The damping force is proportional to the speed, $-b\dot{x}$. An external force $F_0 \sin \omega t$ forces the mass to oscillate back and forth with angular frequency ω.



- Show that the motion of the mass is determined by the equation

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = A_0 \sin \omega t,$$

and thus determine δ, ω_0 and A_0. Here, x is the displacement of the mass relative to its equilibrium position $x = 0$.

- The solution of this equation is

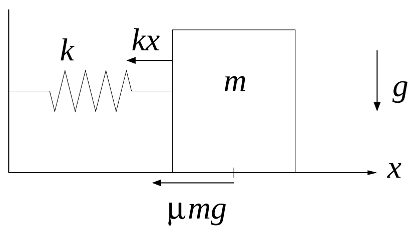
$$x(t) = x_0(\omega)\cos(\omega t + \phi)$$

where the frequency dependence of the amplitude is given by

$$x_0(\omega) = \frac{A_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\omega\delta)^2}}$$

Assume weak damping, i.e. $\delta \ll \omega_0$, and find the full width at half maximum, $\Delta\omega$, of the resonance peak, i.e., the difference between the two angular frequencies $\omega_\pm = \omega_0 \pm \Delta\omega/2$ that both result in an amplitude x_0 equal to the maximum amplitude divided by $\sqrt{2}$. (With weak damping, maximum amplitude is obtained for $\omega \simeq \omega_0$.)

Now, the speed dependent damping is replaced by ordinary friction against a surface with friction coefficient μ (the same both static and dynamic), so that there is a frictional force μmg, with direction opposite to the motion of the mass. Here, g is the acceleration due to gravity. (If the mass is at rest, the frictional force is *up to* μmg, with direction opposite to other forces trying to make the mass move.) We also turn off the external force and study free oscillations. The figure shows, as an example, the situation with $x > 0$ and $\dot{x} > 0$.



Position $x = 0$ for the mass corresponds to a spring which is not stretched or compressed. At time $t = 0$, the mass is pulled to the right, to the position $x(0) = 4.5\mu mg/k$, and released with zero initial velocity.

• Show that the motion of the mass is described by the equation(s)

$$\ddot{\xi} + \omega^2 \xi = 0$$

where we have introduced $\xi = x \pm \mu mg/k$, and where the sign depends on whether the mass moves to the right or to the left. What is the angular frequency ω?

• Sketch $x(t)$ from $t = 0$ to $t = 3T/2$. Here, $T = 2\pi/\omega$ is the period of the oscillation. Give a short comment to the graph.

OPPGAVE 2 [Counts 15%]

In this exercise, we study surface waves on deep water. These waves have the dispersion relation

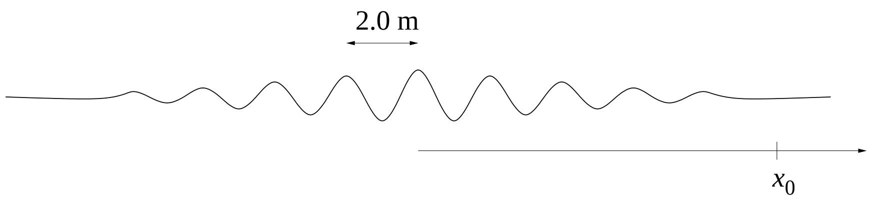
$$\omega(k) = \sqrt{gk + \gamma k^3/\rho}$$

Here, k is the wave number, $g = 9.8$ m/s^2 the acceleration due to gravity, $\rho = 10^3$ kg/m^3 the mass density of water, and $\gamma = 0.073$ N/m the surface tension of water.

• Find the period of waves with wave length $\lambda = 2.0$ m.

• Find the wave length*s* of waves with phase velocity $v_f = 1.0$ m/s.

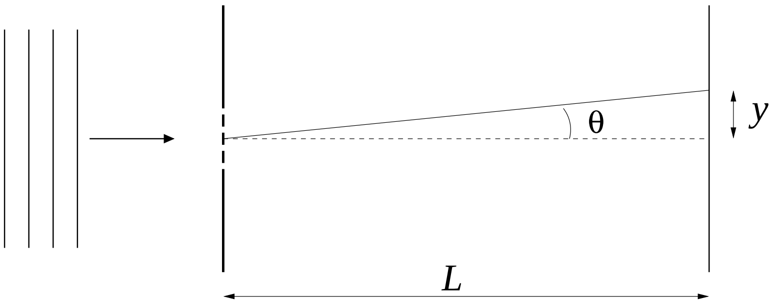
• A wave packet has 9 clear crests and wave length 2.0 m. We assume that the total length of the wave packet is 8.5 wave lengths, i.e. 17.0 m, and that the total length does not change much during our observations. The figure shows a snapshot of the wave packet:



We focus on a fixed position x_0 and watch carefully as the wave packet passes by. How long time does the wave packet spend in passing x_0? How many crests do we observe passing x_0? Explain your answer. (Comment: Such a spatially restricted wave packet "contains" more wave lengths than the mentioned one of 2.0 m. However, the information that the length of the packet does not change during the observations, is sufficient to answer the questions.)

OPPGAVE 3 [Counts 15%]

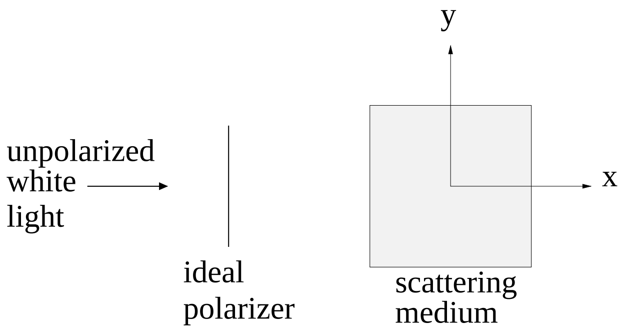
Monochromatic light with wave length $\lambda = 632.8$ nm falls perpendicularly on 4 narrow parallel slits. Each slit has width $a = 40.00$ μm, and the center-to-center distance between the slits is $d = 125.0$ μm. The light that passes through the slits falls on a screen located at a distance of $L = 300.0$ cm from the slits, and results in an intensity pattern $I(y)$. In the figure, the length direction of the slits is perpendicular to the paper plane:



• Write down an expression for the intensity pattern $I(y)$ which is valid for small angles of deviation θ. In this expression, only y should remain as variable parameter (in addition to the prefactor I_0, which represents 1/16 of the intensity in $y = 0$). Use 4 digits for numerical factors in your expression, and state the unit you have chosen for y (m, cm, mm ..).

• How many zeros does $I(y)$ have all together inside the central diffraction maximum? (By "central diffraction maximum" is meant the region between y_0 and $-y_0$, where y_0 corresponds to the position of the first zero if we had only *one* slit of width a.) Sketch $I(y)$ for $-y_0 < y < y_0$.

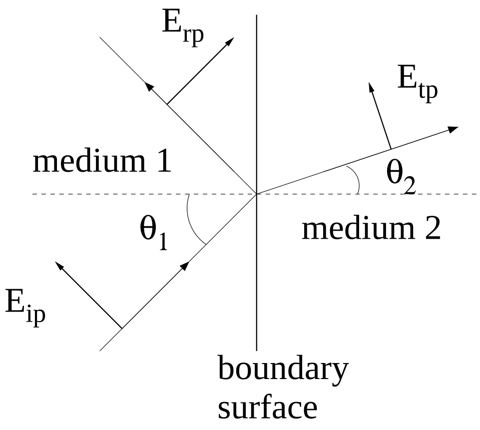
OPPGAVE 4 [Counts 10%]



Unpolarized white light is directed towards a tank containing a scattering medium. The scatterers in the tank may be regarded as induced oscillating electric dipoles with dipole moment $\boldsymbol{p}$ in the same direction as $\boldsymbol{E}$ of the incident light. (The concentration of scatterers is so small that we may disregard the possibility of scattered light being scattered again.) The incident light passes through an ideal polarizer, so that the light striking the tank has $\boldsymbol{E} \parallel \hat{y}$.

• Discuss qualitatively (and briefly) the intensity, the polarization, and the color of the light being scattered in both the y and the z direction.

OPPGAVE 5 [Counts 10%]



A monochromatic plane electromagnetic wave falls on a plane boundary between two nonmagnetic dielectric media 1 and 2, see figure above. If the wave is polarized parallel to the plane of incidence, the relation between the amplitude of the reflected wave, E_{rp0}, and the amplitude of the incident wave, E_{ip0}, is

$$E_{rp0} = \frac{\alpha - \beta}{\alpha + \beta}\, E_{ip0}$$

Here, $\alpha \equiv \cos\theta_2 / \cos\theta_1$ and $\beta \equiv \mu_1 v_1 / \mu_2 v_2$, where μ_j and v_j are magnetic permeability and speed velocity, respectively, in medium nr j $(j = 1, 2)$. With nonmagnetic media, we have $\mu_1 = \mu_2 = \mu_0$. The incident wave is completely transmitted if the angle of incidence θ_1 equals the Brewster angle θ_B.

• Show that the Brewster angle is given by

$$\tan\theta_B = \frac{n_2}{n_1},$$

where n_1 and n_2 are index of refraction in medium 1 and 2, respectively.

OPPGAVE 6 [Counts 10%]

• Show that an electromagnetic wave in vacuum, with

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \sin(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)$$

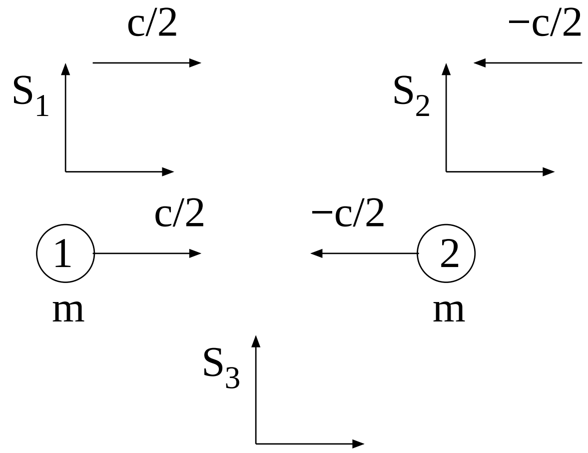
and

$$\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_0 \sin(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)$$

is transversal. Hint: Maxwell's equations.

OPPGAVE 7 [Counts 20%]

Two equal masses m move straight towards each other. Before they collide, the masses 1 and 2 have velocity $c/2$ and $-c/2$, respectively, as measured in the lab system S_3. Here, c is the speed of light. (In other words, we have chosen positive velocities to the right.) The masses 1 and 2 are, prior to collision, at rest in the inertial frames S_1 and S_2, respectively.

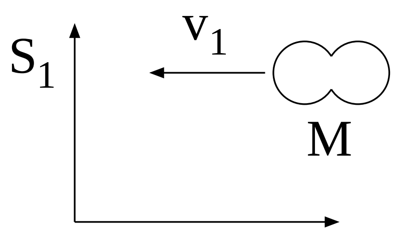


• Before the collision, the masses 1 and 2 are spherically shaped in their rest frames S_1 and S_2, respectively. What is their shape in S_3? Explain your answer.

• Let us with v_{ij} denote the velocity of S_i as observed in S_j $(i, j = 1, 2, 3)$, so that $v_{13} = c/2$ and $v_{23} = -c/2$. Determine the other relative velocities v_{31}, v_{32}, v_{12}, and v_{21}.

• What is the total (relativistic) momentum p_3 and energy E_3 of this system in S_3? (With "system", we simply mean the two masses 1 and 2.) Also determine the total momentum p_1 and energy E_1 in S_1.

The two masses collide, completely inelastically, and stick together as a single body with mass M after the collision.



• Use the principles of momentum and energy conservation to determine M. What is the velocity v_1 of this body as measured in S_1? (S_1 is still the inertial reference frame moving with velocity $c/2$ relative to the lab system S_3.)

Formulas

Bold symbols denote vectors. Symbols with hat above are unit vectors. Validity of formulas and meaning of symbols are assumed to be known.

- Harmonic plane wave:

$$\xi(x,t) = \xi_0 \sin(kx - \omega t + \phi)$$

$$\xi(\boldsymbol{r},t) = \xi_0 \sin(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \phi)$$

- Wave equation:

$$\frac{\partial^2 \xi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \xi(x,t)}{\partial t^2}$$

$$\nabla^2 \xi(\boldsymbol{r},t) \left(\equiv \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} \right) = \frac{1}{v^2}\frac{\partial^2 \xi(\boldsymbol{r},t)}{\partial t^2}$$

- Phase velocity:

$$v = \frac{\omega}{k}$$

- Group velocity:

$$v_g = \frac{d\omega}{dk}$$

- General, for nondispersive undamped waves:

$$v = \sqrt{\frac{\text{elastic modulus}}{\text{massdensity}}}$$

- General, for linear response in elastic media:

$$\text{mechanical stress} = \text{elastic modulus} \times \text{relative strain}$$

- For transverse waves on string:

$$v = \sqrt{\frac{S}{\mu}}$$

- For longitudinal waves in fluids:

$$v = \sqrt{\frac{B}{\rho}}$$

- For longitudinal waves in solids:

$$v = \sqrt{\frac{Y}{\rho}}$$

- Average of harmonically varying $A(x,t)$, averaged over wave length λ:

$$\overline{A} = \frac{\int_0^\lambda A(x,t)\,dx}{\int_0^\lambda dx} = \frac{1}{\lambda}\int_0^\lambda A(x,t)\,dx$$

  Average of harmonically varying $A(x,t)$, averaged over period T:

$$\langle A \rangle = \frac{\int_0^T A(x,t)\,dt}{\int_0^T dt} = \frac{1}{T}\int_0^T A(x,t)\,dt$$

- Average energy pr unit length for harmonic wave on string:

$$\overline{\varepsilon} = \frac{1}{2}\mu\omega^2\xi_0^2$$

- Average energy pr unit volume for harmonic plane wave:

$$\overline{\varepsilon} = \frac{1}{2}\rho\omega^2\xi_0^2$$

- Average power transported with harmonic wave on string:

$$\overline{P} = v\overline{\varepsilon} = \frac{1}{2}v\mu\omega^2\xi_0^2$$

- Average intensity in harmonic plane wave:

$$I = v\overline{\varepsilon} = \frac{1}{2}v\rho\omega^2\xi_0^2$$

- Average momentum density for harmonic wave:

$$\overline{\pi} = \frac{\overline{\varepsilon}}{v}$$

- Ideal gas:

$$pV = Nk_BT$$

- Heat capacity at constant pressure (Q = heat):

$$C_p = \left(\frac{dQ}{dT}\right)_p$$

- Heat capacity at constant volume (Q = heat):

$$C_V = \left(\frac{dQ}{dT}\right)_V$$

- Adiabatic conditions (i.e. no heat exchange):

$$pV^\gamma = \text{constant}$$

- The adiabatic constant:

$$\gamma = \frac{C_p}{C_V}$$

  Gas with 1-atomic molecules: $\gamma = 5/3$. Gas with 2-atomic molecules: $\gamma = 7/5$.

- Bulk modulus for ideal gas at adiabatic conditions:

$$B = \gamma p$$

- Sound velocity in gas (m = molecular mass):

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma k_B T}{m}}$$

- Sound pressure:

$$\Delta p = -B\frac{\partial \xi}{\partial x}$$

- Sound level:

$$\beta(\text{dB}) = 10 \log \frac{I}{I_0}$$

  with $I_0 = 10^{-12}$ W/m^2

- Doppler effect:

$$\nu_\text{O} = \frac{1 - v_\text{O}/v}{1 - v_\text{S}/v}\,\nu_\text{S}$$

- For shock waves:

$$\sin\alpha = \frac{v}{v_\text{S}}$$

- Transverse wave on string with mass density μ_1 for $x < 0$ and μ_2 for $x > 0$, incident wave propagating in positive x direction:

  Amplitude for reflected wave:
  $$y_{r0} = \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}\, y_{i0}$$
  Amplitude for transmitted wave:
  $$y_{t0} = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}\, y_{i0}$$
  Reflection coefficient:
  $$R = \frac{\overline{P}_r}{\overline{P}_i}$$
  Transmission coefficient:
  $$T = \frac{\overline{P}_t}{\overline{P}_i}$$

- Plane sound wave with normal incidence on boundary in $x = 0$ between two media with elastic moduli and mass densities E_1, ρ_1 (for $x < 0$) and E_2, ρ_2 (for $x > 0$), respectively. Incident wave propagating in positive x direction:

  Amplitude for reflected wave:
  $$\xi_{r0} = \frac{\sqrt{\rho_2 E_2} - \sqrt{\rho_1 E_1}}{\sqrt{\rho_2 E_2} + \sqrt{\rho_1 E_1}}\, \xi_{i0}$$
  Amplitude for transmitted wave:
  $$\xi_{t0} = \frac{2\sqrt{\rho_1 E_1}}{\sqrt{\rho_2 E_2} + \sqrt{\rho_1 E_1}}\, \xi_{i0}$$
  Reflection coefficient:
  $$R = \frac{\overline{P}_r}{\overline{P}_i}$$
  Transmission coefficient:
  $$T = \frac{\overline{P}_t}{\overline{P}_i}$$

- Maxwell's equations on integral form:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{A} = q/\varepsilon_0$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt}\int \boldsymbol{B} \cdot d\boldsymbol{A}$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0\varepsilon_0 \frac{d}{dt}\int \boldsymbol{E} \cdot d\boldsymbol{A}$$

- Maxwell's equations on differential form:

$$\nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \mu_0\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

- Lorentz force:

$$\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

- Wave equation for $\boldsymbol{E}$ and $\boldsymbol{B}$ in vacuum:

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\nabla^2 \boldsymbol{B} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2}$$

$$c = 1/\sqrt{\varepsilon_0\mu_0}$$

- Energy density in electromagnetic field:

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

- Intensity in electromagnetic wave:

$$I = c\varepsilon_0 \overline{E^2} = c\varepsilon_0 \langle E^2 \rangle$$

- Poynting's vector:

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}$$

- Momentum in electromagnetic wave:

$$\boldsymbol{\pi} = \mu_0\varepsilon_0 \boldsymbol{S}$$

- Electric dipole moment:
$$\boldsymbol{p} = q\boldsymbol{d}$$
- Magnetic dipole moment:
$$\boldsymbol{m} = I\boldsymbol{A}$$
- Average radiated effect from oscillating electric dipole $p_0 \cos(\omega t)$:
$$\langle P \rangle = \frac{p_0^2 \omega^4}{12\pi\varepsilon_0 c^3}$$
- Average radiated effect from oscillating magnetic dipole $m_0 \cos(\omega t)$:
$$\langle P \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$
- Malus' law:
$$I(\theta) = I_0 \cos^2 \theta$$
- Linear media:
$$\begin{aligned} \boldsymbol{P} &= \varepsilon_0 \chi_e \boldsymbol{E} \\ \boldsymbol{D} &= \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0(1+\chi_e)\boldsymbol{E} = \varepsilon_0\varepsilon_r \boldsymbol{E} = \varepsilon \boldsymbol{E} \\ \boldsymbol{M} &= \chi_m \boldsymbol{H} \\ \boldsymbol{B} &= \mu_0 \boldsymbol{H} + \boldsymbol{M} = \mu_0(1+\chi_m)\boldsymbol{H} = \mu_0\mu_r \boldsymbol{H} = \mu \boldsymbol{H} \end{aligned}$$
- Maxwell's equations etc:
$$\begin{aligned} \oint \boldsymbol{D} \cdot d\boldsymbol{A} &= q_{\text{free}} \\ \oint \boldsymbol{B} \cdot d\boldsymbol{A} &= 0 \\ \oint \boldsymbol{E} \cdot d\boldsymbol{l} &= -\frac{d}{dt} \int \boldsymbol{B} \cdot d\boldsymbol{A} \\ \oint \boldsymbol{H} \cdot d\boldsymbol{l} &= I_{\text{free}} + \frac{d}{dt} \int \boldsymbol{D} \cdot d\boldsymbol{A} \\ \nabla \cdot \boldsymbol{D} &= \rho_{\text{free}} \\ \nabla \cdot \boldsymbol{B} &= 0 \\ \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{H} &= \boldsymbol{j}_{\text{free}} + \frac{\partial \boldsymbol{D}}{\partial t} \end{aligned}$$
- Energy density, Poynting's vector:
$$\begin{aligned} u &= \frac{1}{2}\varepsilon E^2 + \frac{1}{2\mu} B^2 \\ \boldsymbol{S} &= \frac{1}{\mu} \boldsymbol{E} \times \boldsymbol{B} \end{aligned}$$

- For electromagnetic waves in media ($q_{\text{free}} = I_{\text{free}} = 0$):

$$\begin{aligned}
\nabla^2 \boldsymbol{E} &= \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \\
\nabla^2 \boldsymbol{B} &= \frac{1}{v^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} \\
v &= \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{n}
\end{aligned}$$

- Boundary conditions ($q_{\text{free}} = I_{\text{free}} = 0$ in the boundary):

$$\begin{aligned}
\Delta D_{\perp} &= 0 \\
\Delta E_{\parallel} &= 0 \\
\Delta B_{\perp} &= 0 \\
\Delta H_{\parallel} &= 0
\end{aligned}$$

- Reflection and refraction:

$$\theta_r = \theta_i$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

- Young's experiment with two narrow slits:

$$I(\theta) = 4 I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

- Diffraction grating with N narrow slits:

$$I(\theta) = I_0\, \frac{\sin^2\left(\frac{N\pi d}{\lambda}\sin\theta\right)}{\sin^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}$$

- Diffraction from one slit:

$$I(\theta) = I(0)\, \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$$

- Diffraction from N slits of width a:

$$I(\theta) = I_0\, \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}\; \frac{\sin^2\left(\frac{N\pi d}{\lambda}\sin\theta\right)}{\sin^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}$$

- Lorentz factor:

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2}$$

- Lorentz transformations ($\overline{S}$ has velocity $\boldsymbol{v} = v\hat{x}$ relative to S):

$$\begin{aligned} \overline{x} &= \gamma\,(x - vt) \\ \overline{y} &= y \\ \overline{z} &= z \\ \overline{t} &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}$$

$$\begin{aligned} x &= \gamma\,(\overline{x} + v\overline{t}) \\ y &= \overline{y} \\ z &= \overline{z} \\ t &= \gamma\left(\overline{t} + \frac{v}{c^2}\overline{x}\right) \end{aligned}$$

- Time dilation:

$$\Delta t = \gamma \Delta \overline{t}$$

- Length contraction:

$$\Delta \overline{x} = \gamma \Delta x$$

- Velocity in S ($\boldsymbol{u} = u_x\hat{x} + u_y\hat{y} + u_z\hat{z}$):

$$u_x = dx/dt$$

$$u_y = dy/dt$$

$$u_z = dz/dt$$

  Velocity in $\overline{S}$ ($\overline{\boldsymbol{u}} = \overline{u}_x\hat{x} + \overline{u}_y\hat{y} + \overline{u}_z\hat{z}$):

$$\overline{u}_x = d\overline{x}/d\overline{t}$$

$$\overline{u}_y = d\overline{y}/d\overline{t}$$

$$\overline{u}_z = d\overline{z}/d\overline{t}$$

- Addition of velocities (all velocities in the same direction):

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + v_{AB}v_{BC}/c^2}$$

- Doppler effect for electromagnetic waves:

$$\overline{\nu} = \nu\left(\frac{c - v}{c + v}\right)^{1/2}$$

- Relativistic momentum:

$$\boldsymbol{p} = \gamma m \boldsymbol{v}$$

- Newton's 2. law:

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$$

- Energy:

$$\begin{aligned} E &= \gamma mc^2 \\ E_0 &= mc^2 \\ E_k &= E - E_0 \\ E^2 &= (pc)^2 + \left(mc^2\right)^2 \end{aligned}$$

- Elastic process: E, $\boldsymbol{p}$, E_k and m conserved.
- Inelastic process: E and $\boldsymbol{p}$ conserved.