Institutt for fysikk, NTNU
FY1002/TFY4160 Bølgefysikk
Høst 2007

Midterm Bølgefysikk
Friday October 12 2007 at 1215 – 1400.

Mark your answers on page 13. Remember to fill in your student number. Deliver all 13 pages.

Allowed remedies: C

- K. Rottmann: Matematisk formelsamling. (Or similar.)
- O. Øgrim og B. E. Lian: Størrelser og enheter i fysikk og teknikk or B. E. Lian og C. Angell: Fysiske størrelser og enheter.
- Approved calculator, with empty memory, according to list given by NTNU (HP30S or similar.)
- Formulas in wave physics included on the following pages.

Information:

- The test consists of 25 questions. Each question has one correct and three incorrect alternatives.
- Choose one alternative on each question. More than one alternative or zero alternatives will be considered a wrong answer and gives zero points.
- Some values: Acceleration due to gravity: $g = 9.8$ m/s^2, Boltzmann's constant: $k_B = 1.38 \cdot 10^{-23}$ J/K, Avogadro's number: $N_A = 6 \cdot 10^{23}$, Proton mass: $m_p = 1.67 \cdot 10^{-27}$ kg.
- Symbols are given in italics (e.g. m for mass) while units are given without (e.g. m for meter). Vectors are given with bold letters. Unit vectors are given with a hat above.
- SI-prefixes: G (giga) = 10^9, M (mega) = 10^6, k (kilo) = 10^3, c (centi) = 10^{-2}, m (milli) = 10^{-3}, μ (micro) = 10^{-6}, n (nano) = 10^{-9}, p (pico) = 10^{-12}.
- Various stuff:

$$\sin(\alpha + \pi/4) = [\sin\alpha + \cos\alpha]/\sqrt{2}$$

$$\sin(\alpha + \pi/2) = \cos\alpha$$

$$\int (x+a)^{-1/2}\, dx = 2\,(x+a)^{1/2}$$

Formulas in wave physics

Bold symbols denote vectors. Symbols with a hat above denote unit vectors. The validity of the formulas and the meaning of the symbols are assumed to be known.

- Harmonic plane wave:
$$\xi(x,t) = \xi_0 \sin(kx - \omega t + \phi)$$
- Wave equation:
$$\frac{\partial^2 \xi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \xi(x,t)}{\partial t^2}$$
- Phase velocity:
$$v = \frac{\omega}{k}$$
- Group velocity:
$$v_g = \frac{d\omega}{dk}$$
- General, for nondispersive undamped waves:
$$v = \sqrt{\frac{\text{elastic modulus}}{\text{mass density}}}$$
- General, for linear response in elastic media:
$$\text{mechanical stress} = \text{elastic modulus} \times \text{relative strain}$$
- For transverse waves on string:
$$v = \sqrt{\frac{S}{\mu}}$$
- For longitudinal waves in fluids:
$$v = \sqrt{\frac{B}{\rho}}$$
- For longitudinal waves in solids:
$$v = \sqrt{\frac{Y}{\rho}}$$

- Average energy pr unit length for harmonic wave on string:

$$\overline{\varepsilon} = \frac{1}{2}\mu\omega^2\xi_0^2$$

- Average energy pr unit volume for harmonic plane wave:

$$\overline{\varepsilon} = \frac{1}{2}\rho\omega^2\xi_0^2$$

- Average power transported with harmonic wave on string:

$$\overline{P} = v\overline{\varepsilon} = \frac{1}{2}v\mu\omega^2\xi_0^2$$

- Average intensity in harmonic plane wave:

$$I = v\overline{\varepsilon} = \frac{1}{2}v\rho\omega^2\xi_0^2$$

- Average momentum density for harmonic wave:

$$\overline{\pi} = \frac{\overline{\varepsilon}}{v}$$

- Ideal gas:

$$pV = Nk_BT$$

- Heat capacity at constant pressure (Q = heat):

$$C_p = \left(\frac{dQ}{dT}\right)_p$$

- Heat capacity at constant volume (Q = heat):

$$C_V = \left(\frac{dQ}{dT}\right)_V$$

- Adiabatic conditions (i.e. no heat exchange):

$$pV^\gamma = \text{constant}$$

- The adiabatic constant:

$$\gamma = \frac{C_p}{C_V}$$

  Gas with 1-atomic molecules: $\gamma = 5/3$. Gas with 2-atomic molecules: $\gamma = 7/5$.

- Bulk modulus for ideal gas at adiabatic conditions:

$$B = \gamma p$$

- Sound velocity in gas (m = molecular mass):

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma k_B T}{m}}$$

- Sound pressure:

$$\Delta p = -B \frac{\partial \xi}{\partial x}$$

- Sound level:

$$\beta(\text{dB}) = 10 \log \frac{I}{I_0}$$

  with $I_0 = 10^{-12}$ W/m^2

- Doppler effect:

$$\nu_\text{O} = \frac{1 - v_\text{O}/v}{1 - v_\text{S}/v}\, \nu_\text{S}$$

- For shock waves:

$$\sin\alpha = \frac{v}{v_\text{S}}$$

- Transverse wave on string with mass density μ_1 for $x < 0$ and μ_2 for $x > 0$, incident wave propagating in positive x direction:

  Amplitude for reflected wave:
  $$y_{r0} = \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}\, y_{i0}$$
  Amplitude for transmitted wave:
  $$y_{t0} = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}\, y_{i0}$$
  Reflection coefficient:
  $$R = \frac{\overline{P}_r}{\overline{P}_i}$$
  Transmission coefficient:
  $$T = \frac{\overline{P}_t}{\overline{P}_i}$$

- Plane sound wave with normal incidence on boundary in $x = 0$ between two media with elastic moduli and mass densities E_1, ρ_1 (for $x < 0$) and E_2, ρ_2 (for $x > 0$), respectively. Incident wave propagating in positive x direction:

  Amplitude for reflected wave:
  $$\xi_{r0} = \frac{\sqrt{\rho_2 E_2} - \sqrt{\rho_1 E_1}}{\sqrt{\rho_2 E_2} + \sqrt{\rho_1 E_1}}\, \xi_{i0}$$
  Amplitude for transmitted wave:
  $$\xi_{t0} = \frac{2\sqrt{\rho_1 E_1}}{\sqrt{\rho_2 E_2} + \sqrt{\rho_1 E_1}}\, \xi_{i0}$$
  Reflection coefficient:
  $$R = \frac{\overline{P}_r}{\overline{P}_i}$$
  Transmission coefficient:
  $$T = \frac{\overline{P}_t}{\overline{P}_i}$$

Questions

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1) A mass is attached to a spring and executes undamped harmonic oscillations with angular frequency ω. At a certain moment, the spring is stretched a distance x_0 and the velocity of the mass is zero. What is the maximum velocity of the mass?

A ωx_0
B $2\omega x_0$
C $\sqrt{2}\omega x_0$
D $\omega x_0/\sqrt{2}$

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2) A mass m is attached to a spring with spring constant k and executes damped oscillations. The frictional force is $b \cdot v$, where v is the velocity of the mass and b is a damping constant. Provided we have *weak* damping, the mass will oscillate with period

$$T = \frac{2\pi}{\sqrt{k/m - b^2/4m^2}}$$

Then, what is the oscillation period for the charge q on the capacitor (and the current I) in an electric circuit consisting of a resistance R, a capacitance C, and an inductance L connected in series (assuming the resistance R is relatively small)?
Given information:

$$L\frac{dI}{dt} + RI + \frac{q}{C} = 0$$

$$I = \frac{dq}{dt}$$

A $T = 4\pi L/\sqrt{1/C - R^2}$
B $T = (2\pi L/R)/\sqrt{1 - L/CR^2}$
C $T = 2\pi/\sqrt{LC - L^2R^2/4}$
D $T = (4\pi L/R)/\sqrt{4L/CR^2 - 1}$

Figure 1:

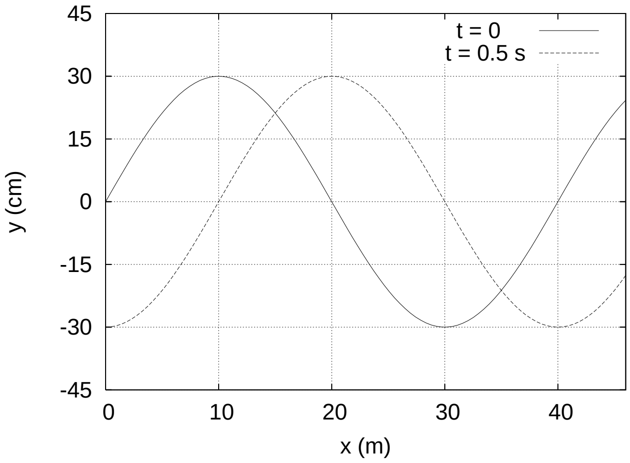

Figure 1 applies to questions 3-6 below, and illustrates two pictures of (parts of) a harmonic transverse wave propagating in the positive x direction on a string.

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3) What can the wave velocity be?

A 10 m/s
B 50 m/s
C 100 m/s
D 150 m/s

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4) What is the corresponding frequency?

A 1.0 Hz
B 1.5 Hz
C 2.0 Hz
D 2.5 Hz

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5) What is the maximum velocity of the string elements?

A 1.9 m/s
B 2.7 m/s
C 3.9 m/s
D 4.7 m/s

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6) The wave may be described with the function $y_0 \sin(kx - \omega t - \phi)$. What is the phase constant ϕ?

A $\phi = 0$
B $\phi = \pi/4$
C $\phi = \pi/2$
D $\phi = 3\pi/4$

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7) What is the sound velocity in hydrogen gas (H_2) at room temperature (300 K)? A hydrogen atom has one proton in the nucleus (and no neutrons).

A 345 m/s
B 928 m/s
C 1317 m/s
D 1641 m/s

8) On a summer day, the temperature suddenly drops from 300 K to 298 K. What is the corresponding change in the sound velocity?

A -0.5 %
B -0.3 %
C $+0.3$ %
D $+0.5$ %

9) A small loudspeaker emits sound waves with equal intensity in all directions. If you measure an intensity level of 80 dB in a distance 40 m from the loudspeaker, what is the intensity level 5 m from the loudspeaker?

A 98 dB
B 104 dB
C 108 dB
D 115 dB

10) The plane wave $\xi(\boldsymbol{r}, t) = \xi_0 \sin(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t + \phi)$ propagates

A in the same direction as $\boldsymbol{r}$.
B in the same direction as $\boldsymbol{k}$.
C in a direction perpendicular to $\boldsymbol{r}$.
D in a direction perpendicular to $\boldsymbol{k}$.

11) The wave

$$\boldsymbol{D}(x, t) = D_0 \hat{y} \sin(kx - \omega t) + 3D_0 \hat{z} \cos(kx - \omega t)$$

is

A unpolarized.
B linearly polarized.
C circularly polarized.
D elliptically polarized.

12) A guitar string of length 75 cm is attached in both ends. The string is stretched with a force 150 N and its mass is 7.2 g. What is the frequency of the 2. harmonic of the string (i.e. the second lowest resonant frequency)?

A 103 Hz
B 119 Hz
C 141 Hz
D 167 Hz

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13) Two cars drive straight away from each other, car nr 1 with velocity 50 m/s and car nr 2 with velocity 5 m/s. Both cars are equipped with a siren that generates a harmonic sound wave with frequency 500 Hz. There is no wind, and the weather is such that the sound velocity this day is $v = 340$ m/s. What frequency ν_1 is measured by car nr 1 from the siren in car nr 2, and what frequency ν_2 is measured by car nr 2 from the siren in car nr 1?

A $\nu_1 = 420$ Hz and $\nu_2 = 420$ Hz
B $\nu_1 = 420$ Hz and $\nu_2 = 429$ Hz
C $\nu_1 = 434$ Hz and $\nu_2 = 425$ Hz
D $\nu_1 = 429$ Hz and $\nu_2 = 429$ Hz

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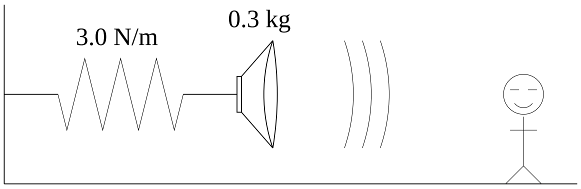
14) A loudspeaker with mass 0.3 kg is attached to an approximately massless spring with spring constant 3.0 N/m. The loudspeaker executes harmonic motion with amplitude 0.15 m and emits a tone of 800 Hz. If you are located right in front of the loudspeaker, you will hear small variations in the frequency of this tone. What is the difference between the highest and the lowest frequency that you will hear? The sound velocity is 340 m/s.

A 1.6 Hz
B 2.2 Hz
C 2.9 Hz
D 3.6 Hz



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15) The shock wave from a supersonic airplane flying horizontally hits you 1.85 s after the airplane passed directly above you. The sound velocity if 340 m/s, and the velocity of the airplane is 2.3 times bigger (i.e. Mach number 2.3). How high above the ground is the airplane flying?

A ca 0.7 km
B ca 1.7 km
C ca 2.7 km
D ca 3.7 km

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16) A long and thin pipe is open in one end and closed in the other end. The pipe will be used to generate standing sound waves of frequency 250 Hz. This is supposed to be the lowest resonant frequency of the pipe. How long should the pipe be? The sound velocity is 340 m/s.

A 34 cm
B 68 cm
C 102 cm
D 136 cm

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17) What is the third lowest resonant frequency of the pipe in the previous question?

A 500 Hz
B 750 Hz
C 1000 Hz
D 1250 Hz

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18) Two equal guitar strings are stretched with slightly different forces so that they oscillate with frequencies 440 and 444 Hz, respectively (however, with equal amplitudes). What do you hear?

A Two tones, 440 and 444 Hz, and constant intensity.
B One tone with frequency 884 Hz, where the intensity varies between strong and weak with period half a second.
C One tone with frequency 442 Hz, where the intensity varies between strong and weak with period a quarter of a second.
D Two tones, 440 and 444 Hz, where the intensity varies between strong and weak with period four seconds.

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19) Two harmonic sound waves with equal amplitude ξ_0 and equal angular frequency ω propagate in the positive x direction. The two sound waves have a phase difference $\pi/2$. With only one of them present, the average intensity would be I_0. Then, what is the average intensity with both sound waves present at the same time?

A I_0
B $\sqrt{2}I_0$
C $2I_0$
D $4I_0$

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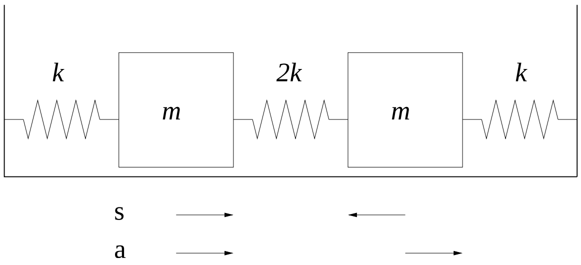
20) Two equal masses m are attached to springs with spring constants k and $2k$ as shown in the figure. The two masses can oscillate in two vibrational modes ("normal modes"): a "symmetric" mode (s) where the masses' displacements from equilibrium are equal but with opposite sign, and an "antisymmetric" mode (a) where the masses' displacements from equilibrium are equal and with the same sign. The corresponding angular frequencies are ω_s and ω_a, respectively. What is the ratio between these two, i.e., ω_s/ω_a?

A $\sqrt{2}$
B $\sqrt{3}$
C $\sqrt{5}$
D $\sqrt{7}$



21) A gaussian wave packet

$$\xi(x,t) = \xi_0 \exp\left[-\frac{(x-vt)^2}{a^2}\right]$$

propagates with velocity v along a string with mass μ pr unit length. Here, $\xi(x,t)$ denotes the transverse displacement of the string. What is the total momentum p of this wave packet?
Given information:

$$p = \int_{-\infty}^{\infty} \mu \frac{\partial \xi}{\partial t}\left(1 - \frac{\partial \xi}{\partial x}\right) dx$$

$$\int_{-\infty}^{\infty} \beta^2 e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{2}$$

$$\int_{-\infty}^{\infty} \beta e^{-\beta^2} d\beta = 0$$

A $\mu\, \xi_0^2\, (v^2/a^2)\sqrt{\pi/2}$
B $\mu\, \xi_0\, v\, \sqrt{\pi}$
C $\mu\, \xi_0\, v\, \sqrt{\pi/2}$
D $\mu\, \xi_0^2\, (v/a)\sqrt{\pi/2}$

22) A steel rod is connected to a copper rod in a smooth and plane surface. Steel has mass density 7800 and Young's modulus $2.0 \cdot 10^{11}$, whereas copper has mass density 8900 and Young's modulus $1.1 \cdot 10^{11}$ (all given in SI units). A harmonic sound wave propagates along the steel rod towards the connecting surface. What is the ratio between the amplitude of the sound wave that is transmitted into the copper rod and the amplitude of the incident sound wave?

A 1.12
B 0.88
C 0.65
D 0.12

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23) You have gone swimming on a lovely summer day. With one ear above and one ear below the water surface, you suddenly hear the sound of an explosion somewhere out on the lake. The sound wave below the water surface is heard two seconds before the sound wave in air. Approximately what is the distance from your position to the location of the explosion? The water has, in SI units, a bulk modulus $2.1 \cdot 10^9$ and a mass density 10^3. The sound velocity in air is 340 m/s.

A 630 m
B 890 m
C 1120 m
D 1460 m

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24) A string of length L hangs vertically in the field of gravity. The upper half has mass μ_1 pr unit length and the lower half has mass $3\mu_1$ pr unit length. A wave pulse is generated near the top of the string and propagates down the string. What is the fraction of the energy of the wave pulse that passes the midpoint of the string and continues down the lower half?

A $2\sqrt{3}/(2+\sqrt{3})$
B $\sqrt{3}/(1+\sqrt{3})$
C $2/(1+\sqrt{3})$
D $1/(2+\sqrt{3})$

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25) How long time will the wave pulse in the previous question spend traveling the full length of the string, from top to bottom? (Hint: The stretching force in a given position is determined by the weight of the mass below.)

A $(3+\sqrt{3})\sqrt{L/g}$
B $(3-\sqrt{3})\sqrt{2L/g}$
C $(6-\sqrt{2})\sqrt{Lg}$
D $(2-\sqrt{3})\sqrt{L/g}$

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FY1002/TFY4160 Bølgefysikk

Midterm Friday October 12 2007 at 1215 – 1400.

Course code: ☐ Student number: ☐

Question	A	B	C	D	Question	A	B	C	D
1	☐	☐	☐	☐	14	☐	☐	☐	☐
2	☐	☐	☐	☐	15	☐	☐	☐	☐
3	☐	☐	☐	☐	16	☐	☐	☐	☐
4	☐	☐	☐	☐	17	☐	☐	☐	☐
5	☐	☐	☐	☐	18	☐	☐	☐	☐
6	☐	☐	☐	☐	19	☐	☐	☐	☐
7	☐	☐	☐	☐	20	☐	☐	☐	☐
8	☐	☐	☐	☐	21	☐	☐	☐	☐
9	☐	☐	☐	☐	22	☐	☐	☐	☐
10	☐	☐	☐	☐	23	☐	☐	☐	☐
11	☐	☐	☐	☐	24	☐	☐	☐	☐
12	☐	☐	☐	☐	25	☐	☐	☐	☐
13	☐	☐	☐	☐					

NB: Make sure you have chosen ONE of the alternatives for each of the 25 questions.