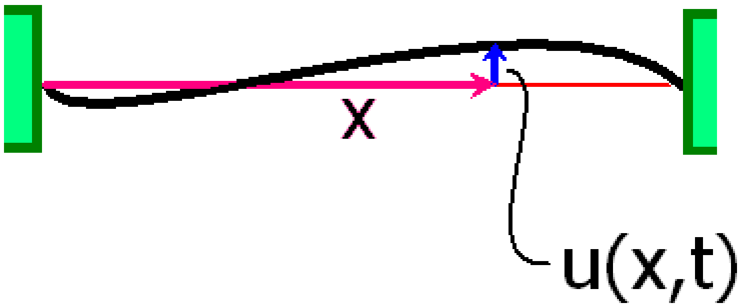
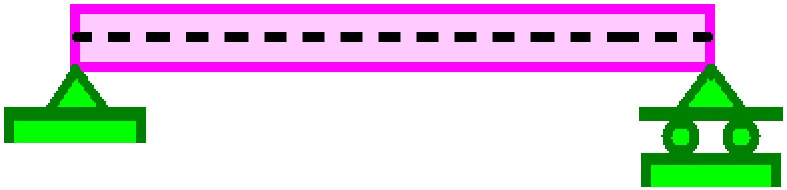
Free vibration of a beam as compared to a taut cable



A “cable” in this context implies no stiffness; the restoring force comes from the fact that the cable is in tension (assumed constant) T.

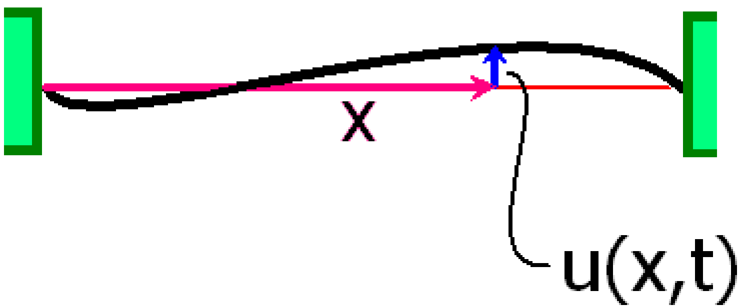


A “beam” on the other hand possesses stiffness, EI, but has no tension.

Free vibration of a beam as compared to a taut cable

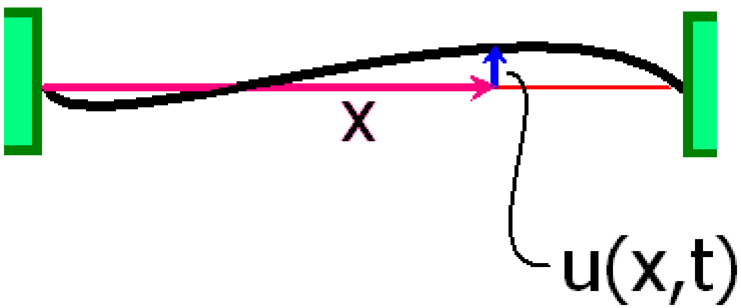
Many structural members in fact possess a combination of tension (compression) *and* stiffness.

Free vibration of a beam as compared to a taut cable



Recall that the vibrating cable or string entailed a second "independent variable" besides time t. This is denoted as "x".

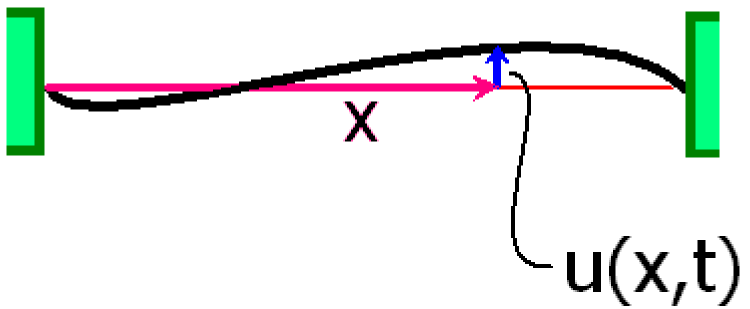
Free vibration of a beam as compared to a taut cable

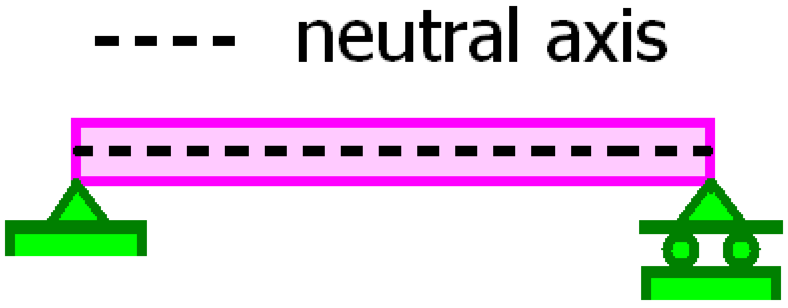


Recall that the vibrating cable or string entailed a second "independent variable" besides time t. This is denoted as "x".

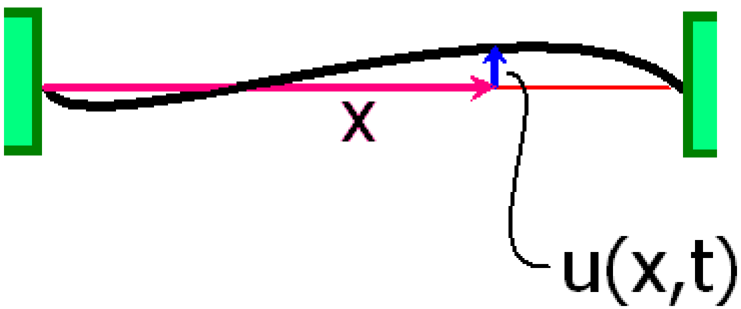
So the deflection is u(x,t).

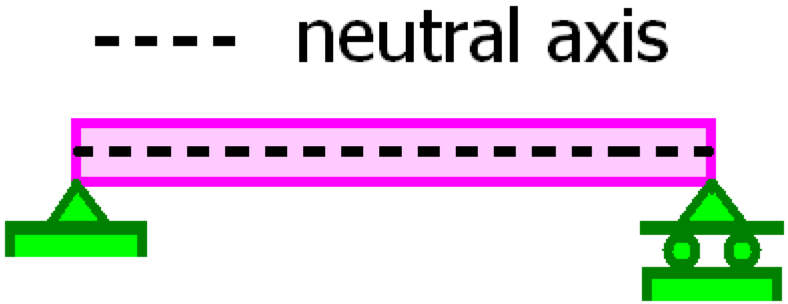
Free vibration of a beam as compared to a taut cable





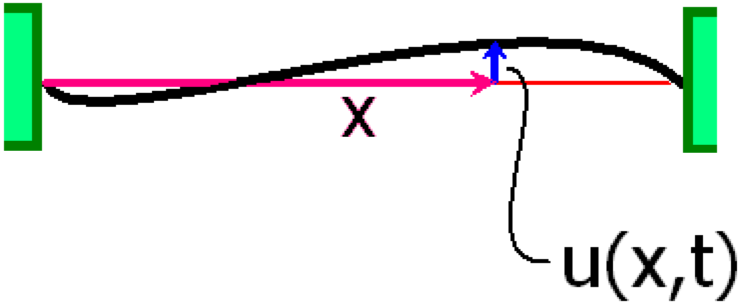
Free vibration of a beam as compared to a taut cable

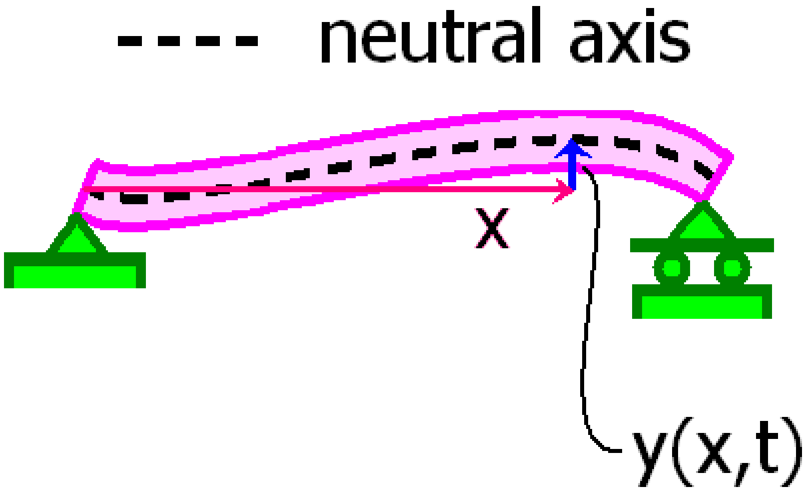




With the beam, we keep track of the deflection of the neutral axis.

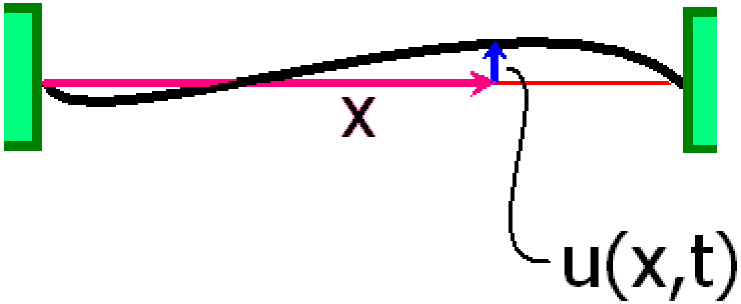
Free vibration of a beam as compared to a taut cable

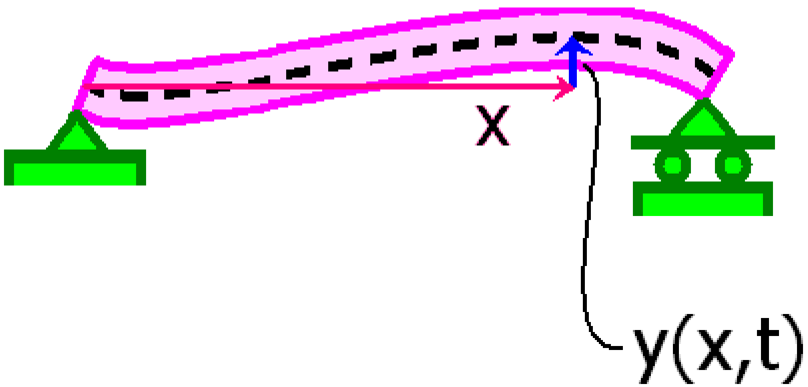




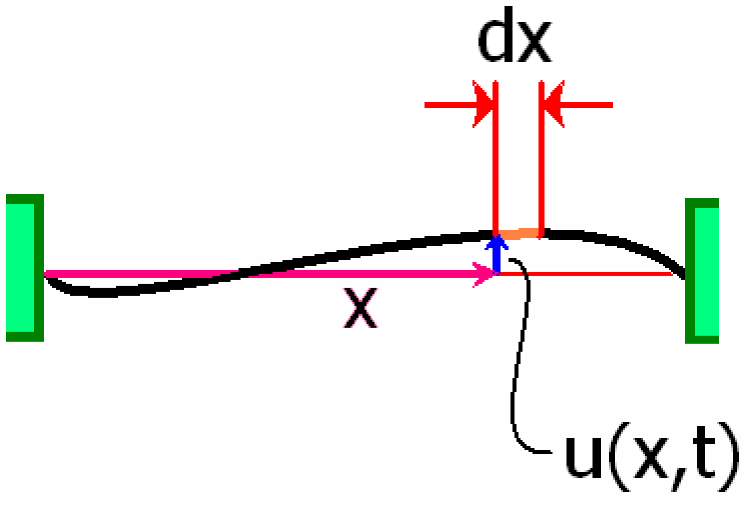
In this case we denote the deflection of the neutral axis by y(x,t).

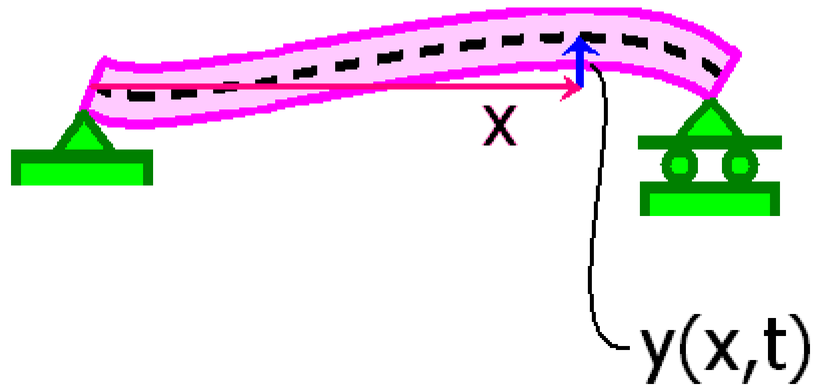
Comparing the governing partial differential equations





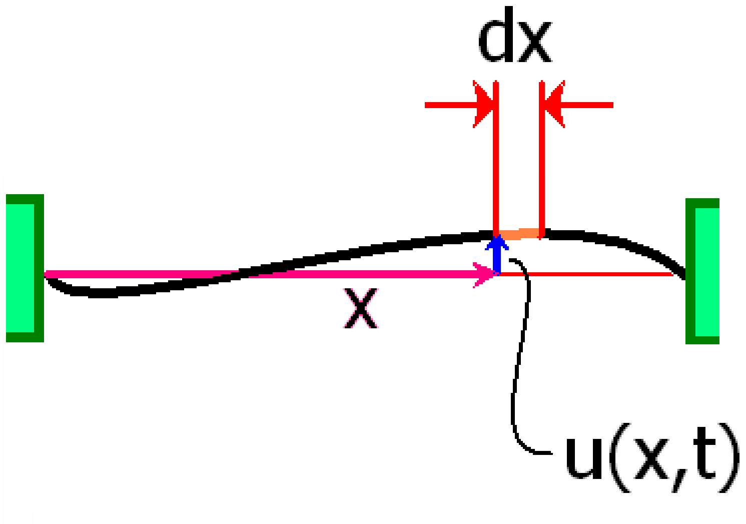
Comparing the governing partial differential equations

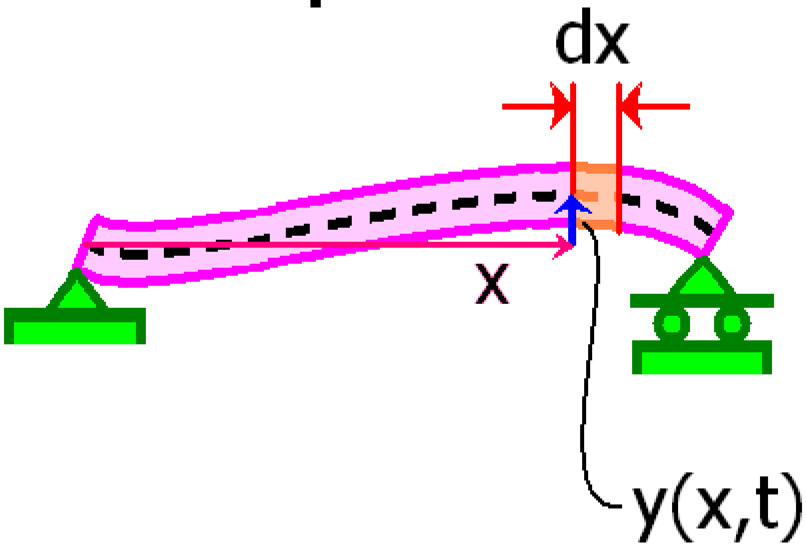




Consider a differential element of the cable, dx.

Comparing the governing partial differential equations

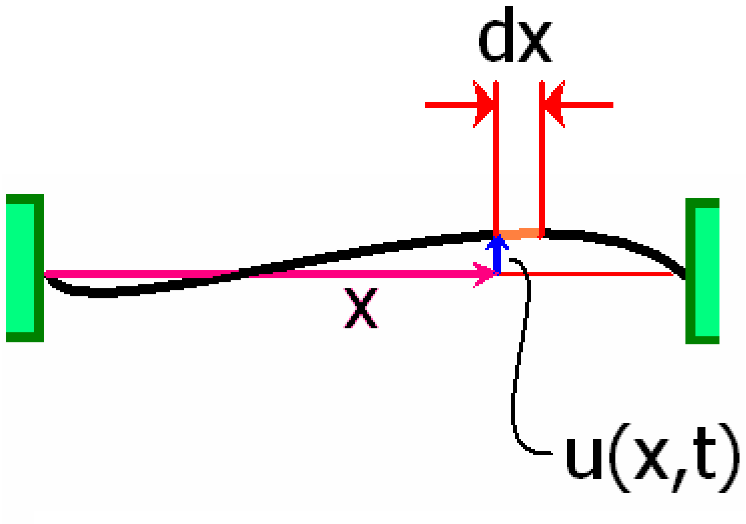




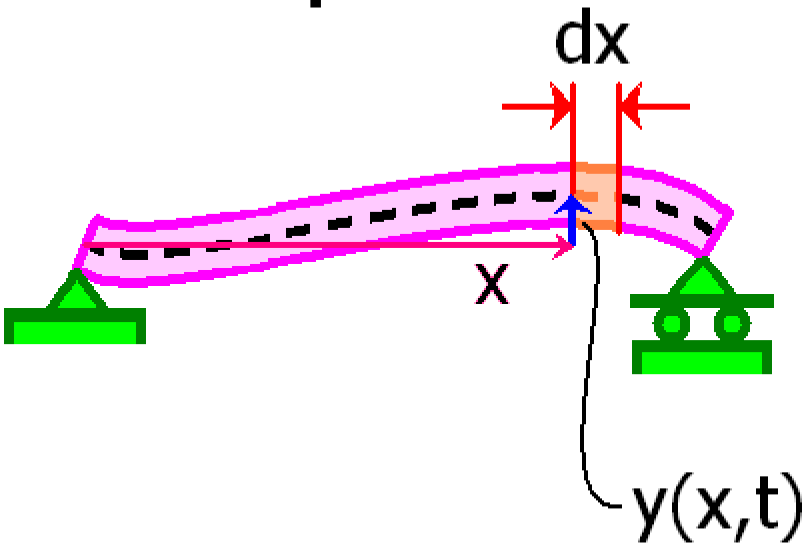
Consider a differential element of the cable, dx.

Also consider a differential element dx of the beam.

Comparing the governing partial differential equations



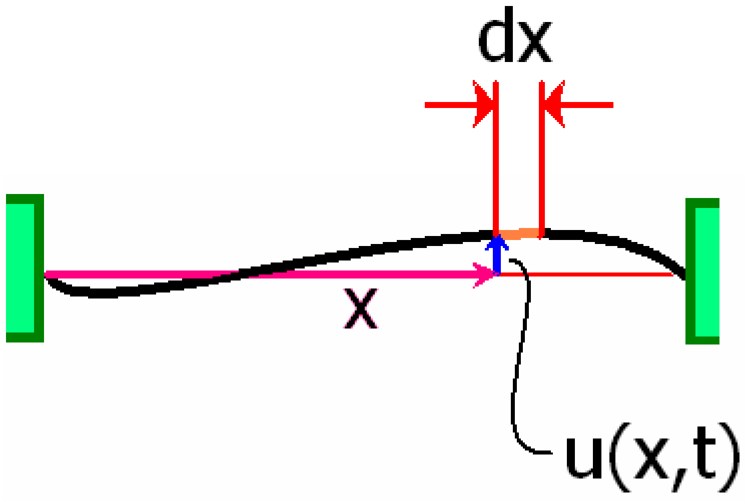
Consider a differential element of the cable, dx.

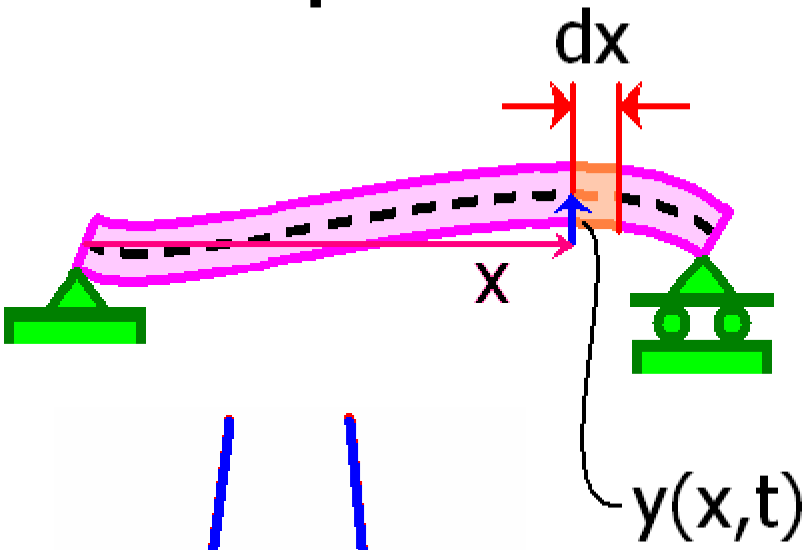


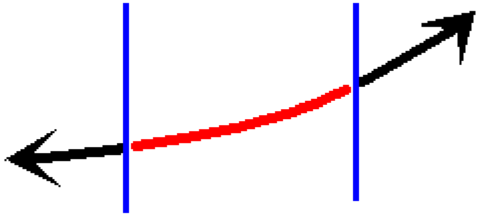
Also consider a differential element dx of the beam.

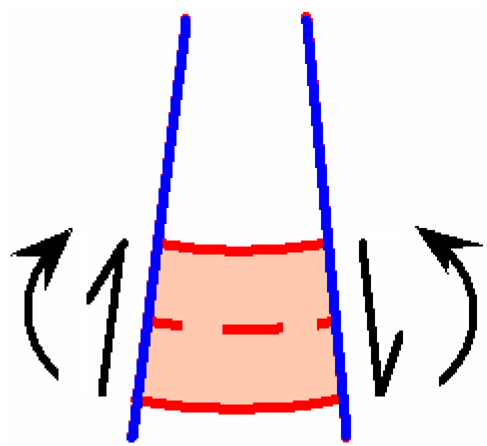
In both cases neglecting gravity, we consider a free body diagram of the two elements.

Comparing the governing partial differential equations



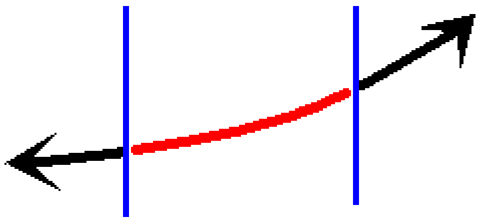


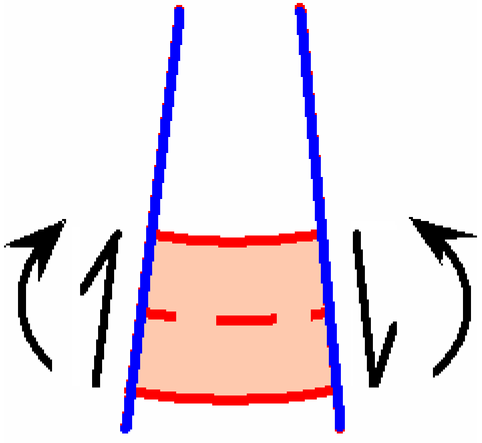




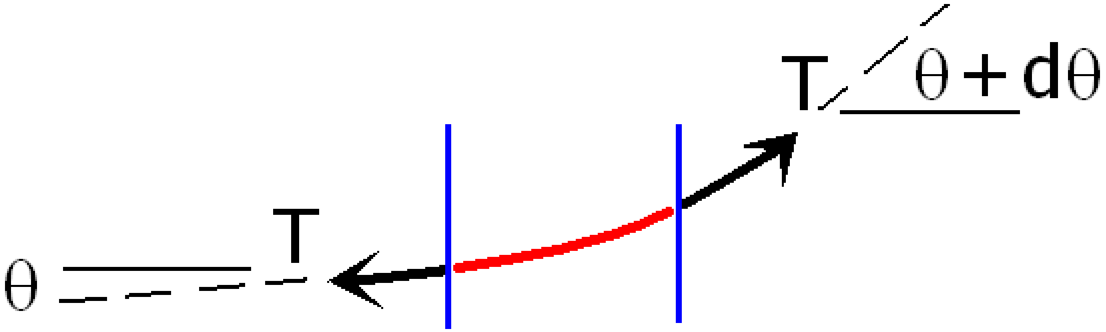
In both cases neglecting gravity, we consider a free body diagram of the two elements.

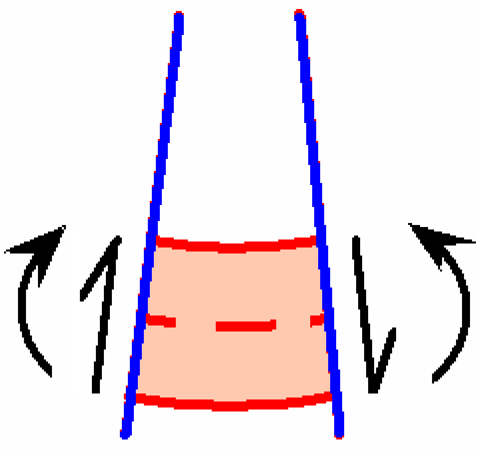
Free body diagrams



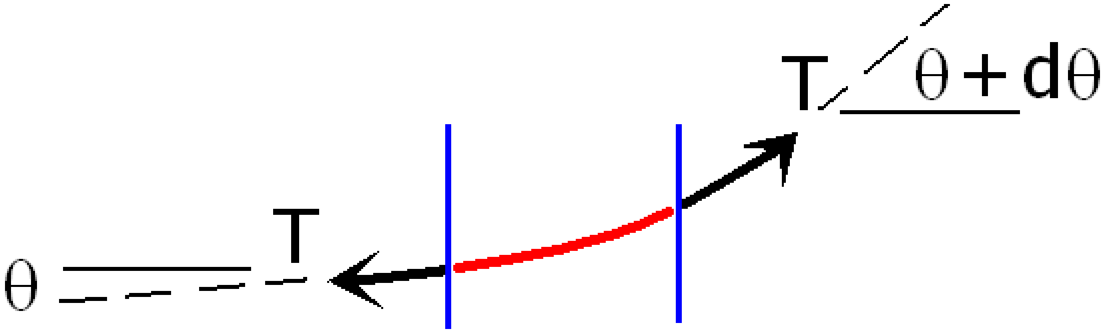


Free body diagrams



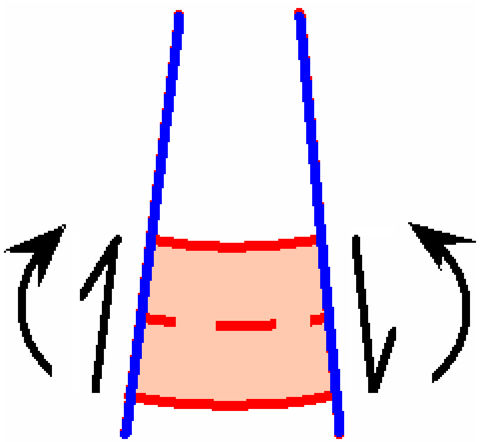


Free body diagrams

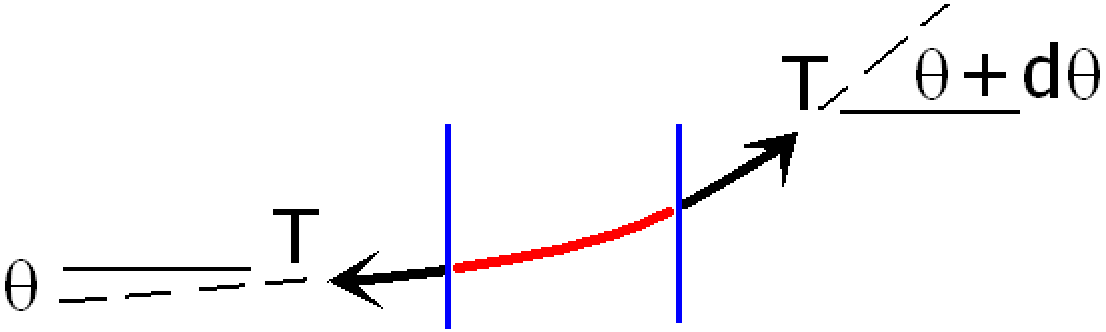


$$\Sigma F_{vert.} = (m/L)\,dx\,\frac{d^2u}{dt^2}$$

$$= -T\sin(\theta) + T\sin(\theta + d\theta)$$



Free body diagrams



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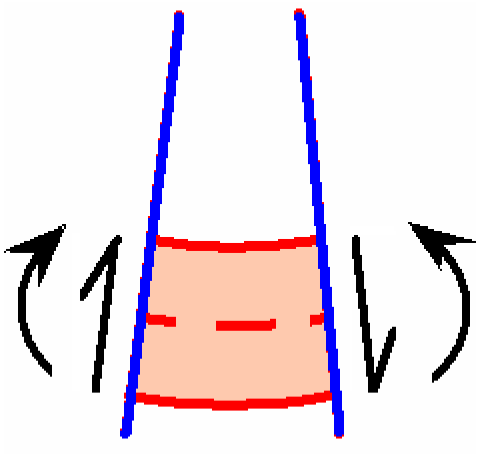
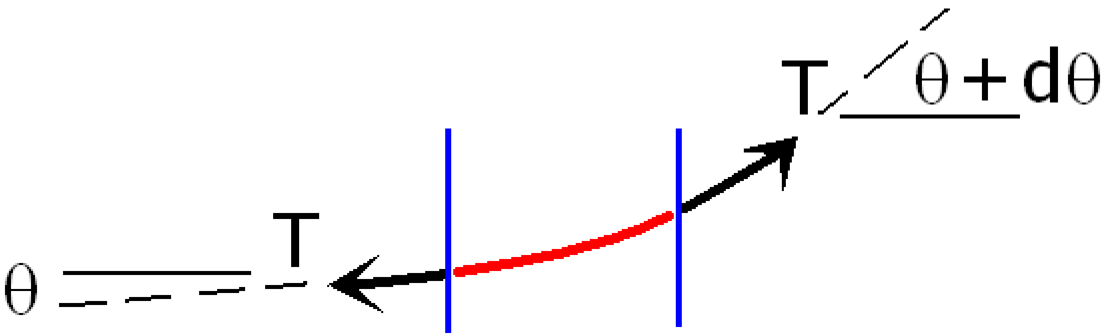


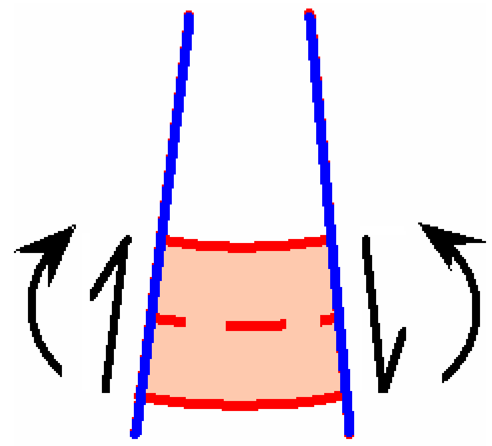
Linearizing about small θ



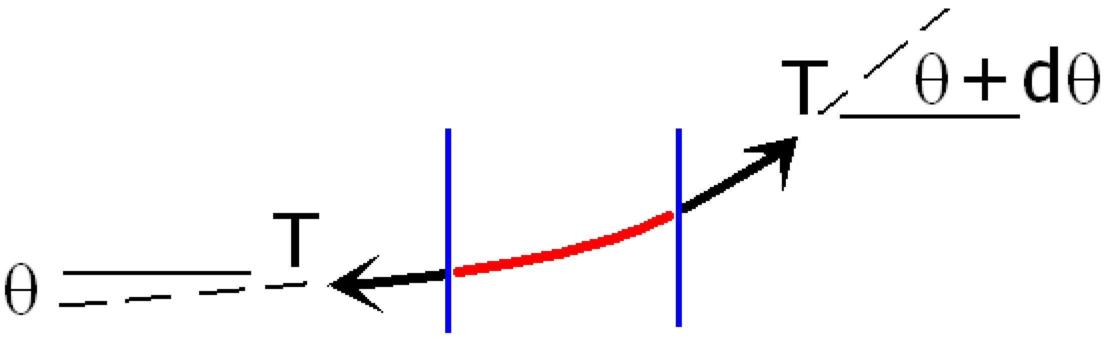
$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 u}{\partial t^2}$$

$$= \; -T\sin(\theta) + T\sin(\theta + \partial\theta)$$

$$\approx T\,\partial\theta$$



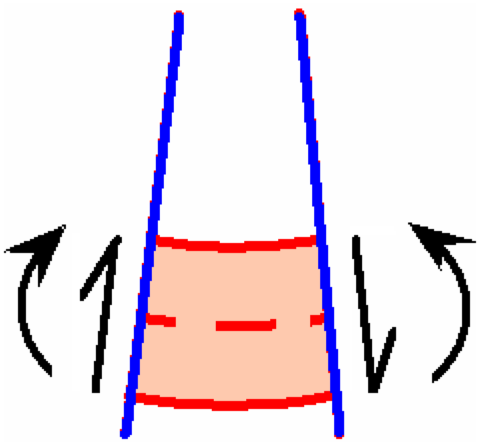
Linearizing about small θ



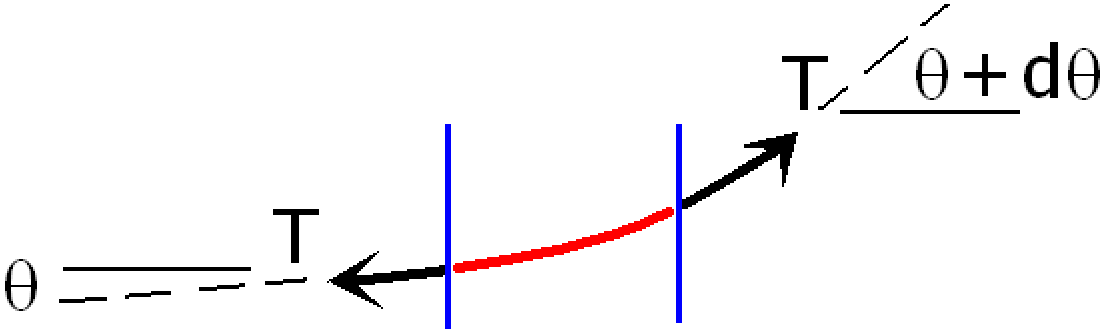
$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 u}{\partial t^2}$$

$$= -T\sin(\theta) + T\sin(\theta + \partial\theta)$$

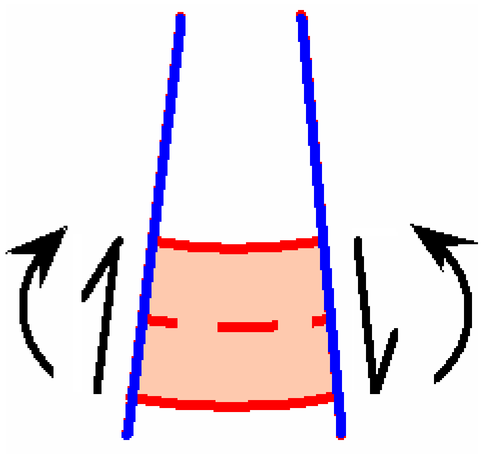
$$\approx T\,\partial\theta \approx T\,\partial\frac{\partial u}{\partial x}$$



Finally, for the cable



$$\frac{\partial^2 u}{\partial t^2} = \frac{TL}{m}\frac{\partial^2 u}{\partial x^2}$$



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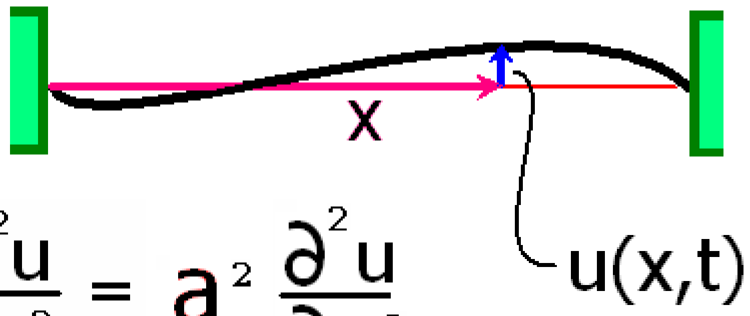
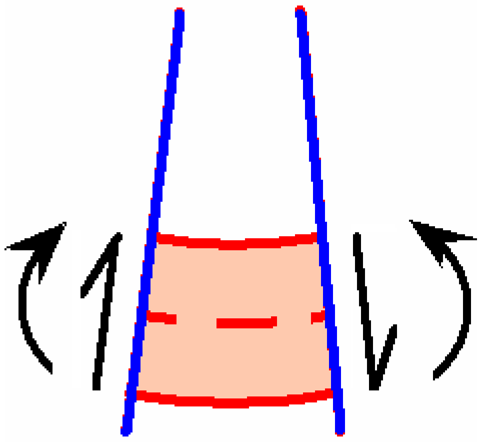
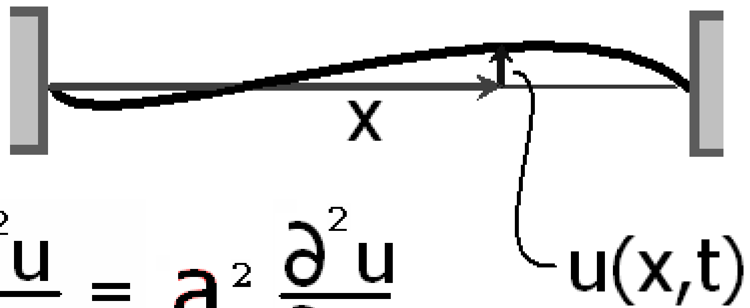


$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

"wave equation" w/
boundary conditions:
u(0,t)=u(L,t)=0



Next, turning to the beam:



$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

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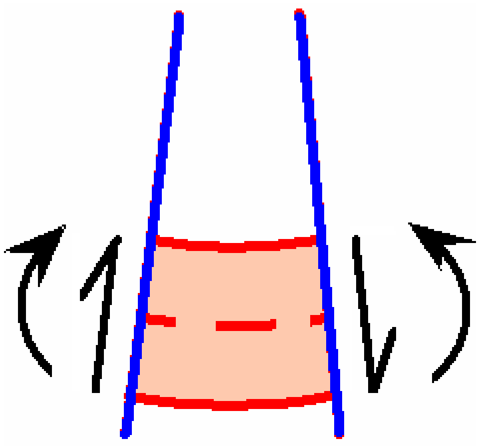

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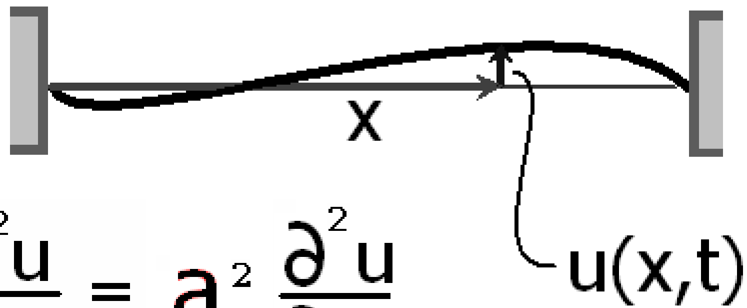

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

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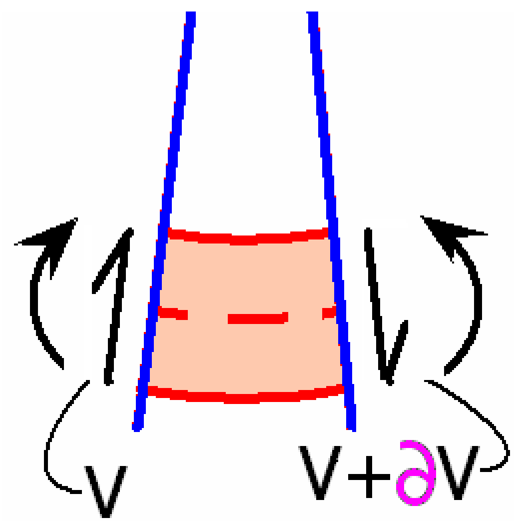
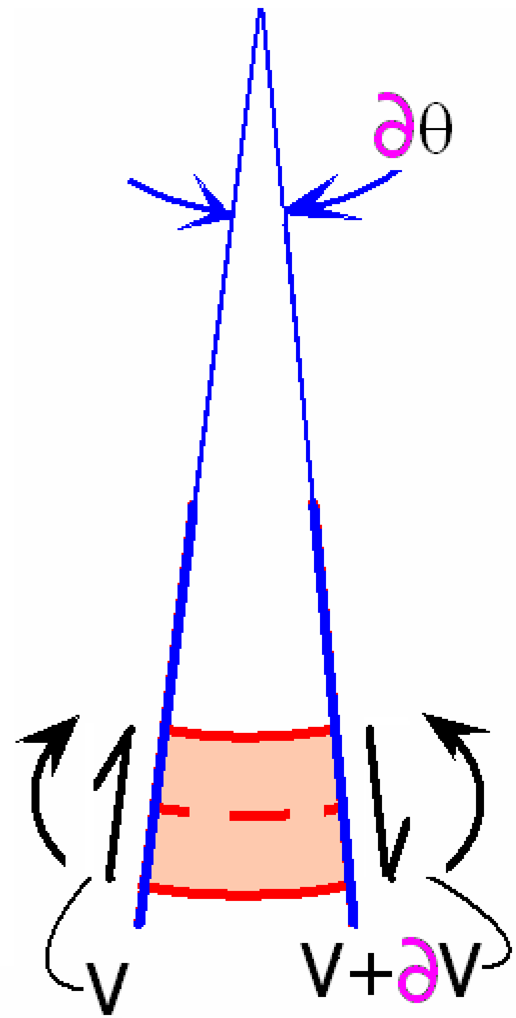


$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 y}{\partial t^2} = -\partial V$$

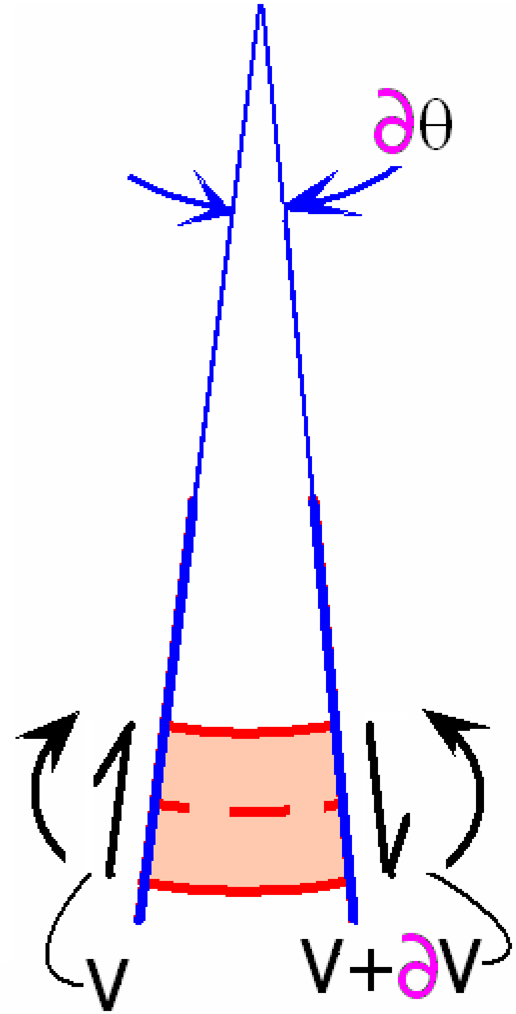
Next, turning to the beam:



$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

$$\rho \partial\theta = \partial x$$



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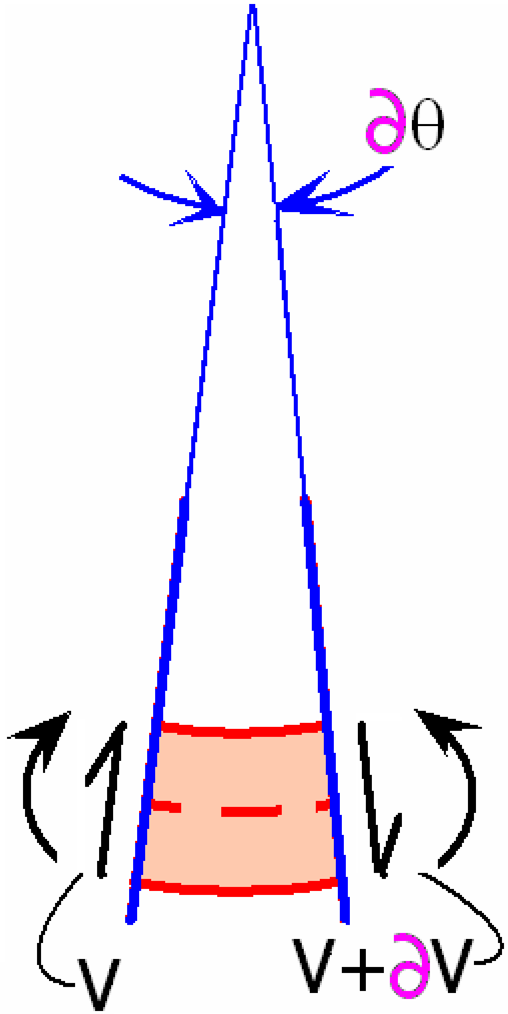
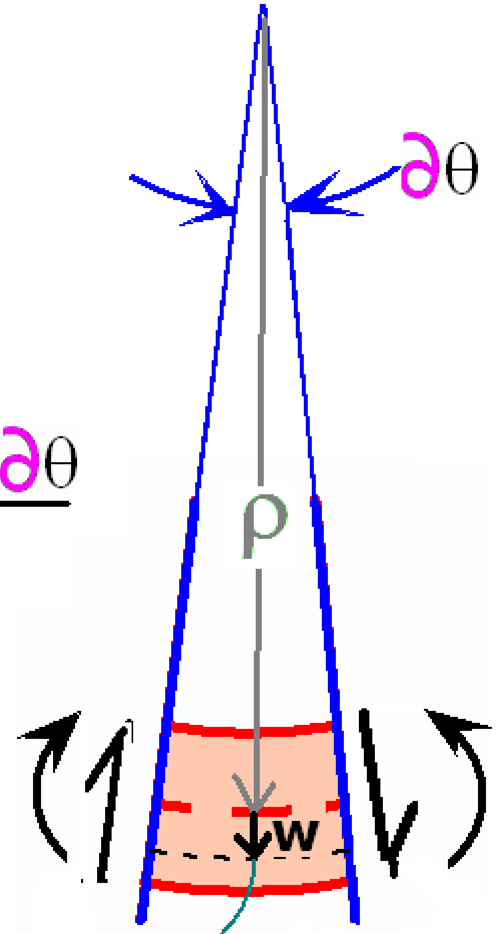


$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

$$\rho \partial\theta = \partial x$$

$$\varepsilon = \frac{(w + \rho)\partial\theta - \rho\partial\theta}{\rho\partial\theta}$$



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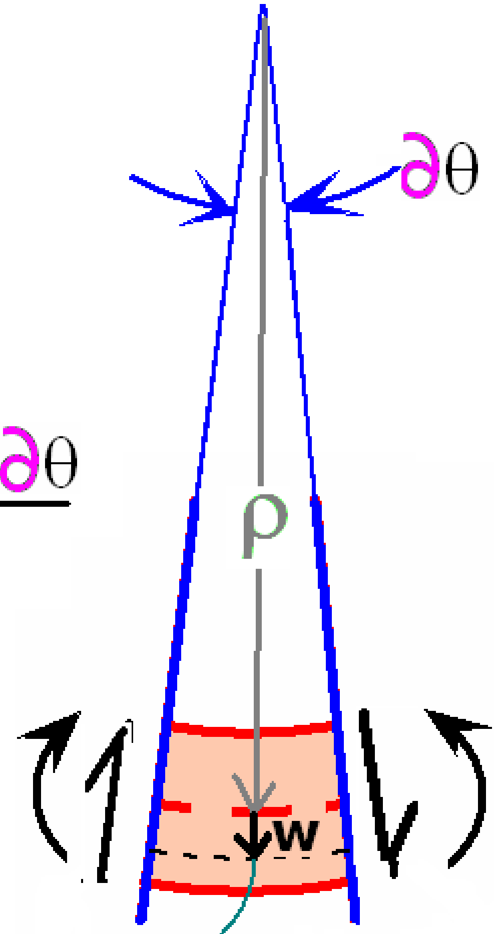
Next, turning to the beam:

$$\rho\,\partial\theta = \partial x$$

$$\varepsilon = \frac{(\mathbf{w}+\rho)\,\partial\theta - \rho\,\partial\theta}{\rho\,\partial\theta}$$

$$= \frac{\mathbf{w}}{\rho}$$



$$\Sigma F_{vert.} = (m/L)\,\partial x\,\frac{\partial^2 y}{\partial t^2} = -\partial V$$

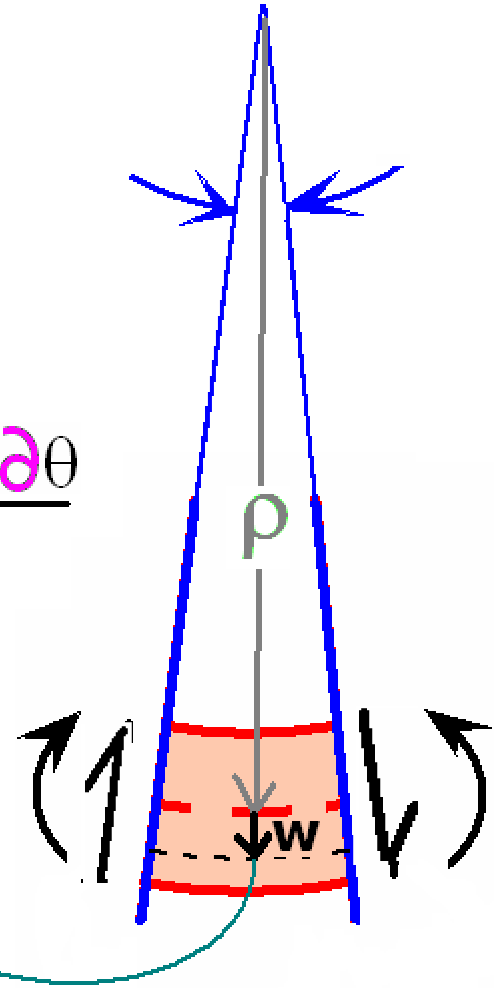
Next, turning to the beam:

$$\rho\partial\theta = \partial x$$

$$\varepsilon = \frac{(w+\rho)\partial\theta - \rho\partial\theta}{\rho\partial\theta}$$

$$= \frac{w}{\rho}$$

$$\sigma = E\varepsilon = E\frac{w}{\rho}$$

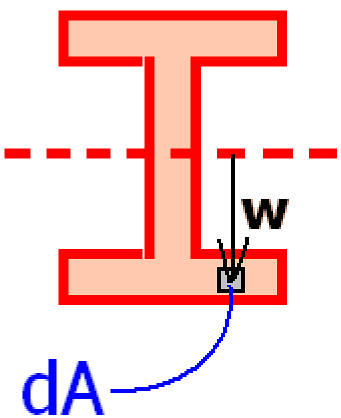


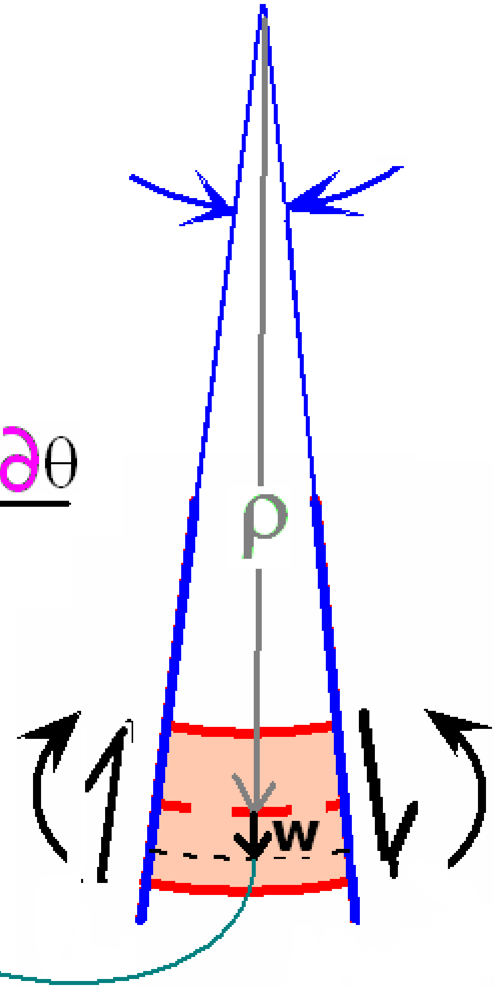
$$\Sigma F_{vert.} = (m/L)\partial x\frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

$$\rho\,\partial\theta = \partial x$$

beam cross section





$$\varepsilon = \frac{(w+\rho)\,\partial\theta - \rho\,\partial\theta}{\rho\,\partial\theta}$$

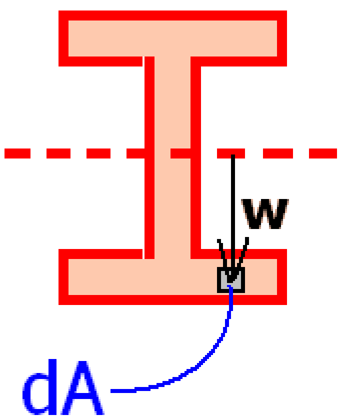
$$= \frac{w}{\rho}$$

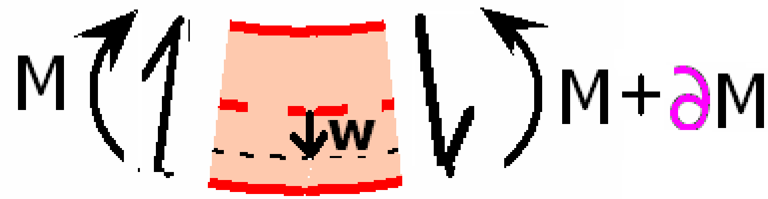
$$\sigma = E\varepsilon = E\,\frac{w}{\rho}$$

$$\Sigma F_{vert.} = (m/L)\,\partial x\,\frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

beam cross section



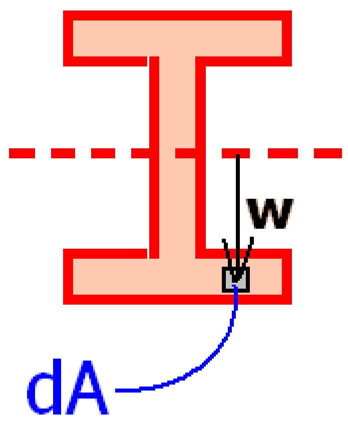


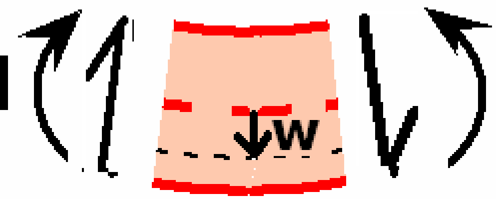
$$\sigma = E\varepsilon = E\frac{w}{\rho}$$

$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

beam cross section



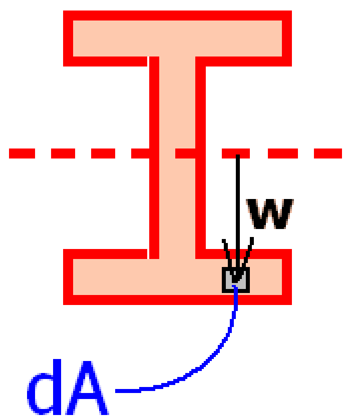
$$\iint_A w\,\sigma\,dA = M$$ 

$M+\partial M$

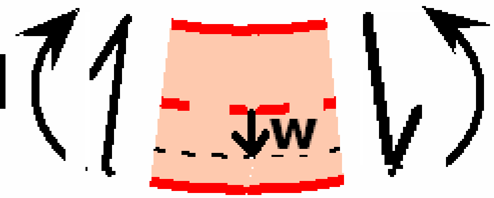
$$\Sigma F_{vert.} = (m/L)\partial x \frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

beam cross section



$$\iint_A w\,\sigma\,dA = M$$



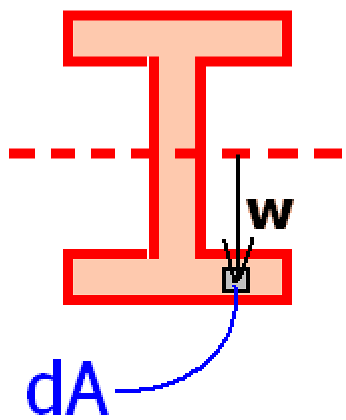
$$M+\partial M$$

$$= \iint_A w\,E\,\frac{w}{\rho}\,dA$$

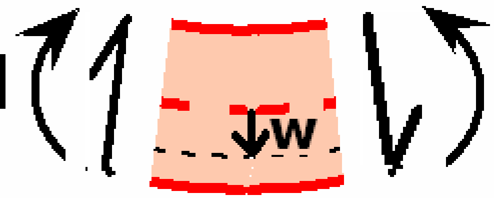
$$\Sigma F_{vert.} = (m/L)\partial x\,\frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

beam cross section



$$\iint_A w\,\sigma\,dA = M$$



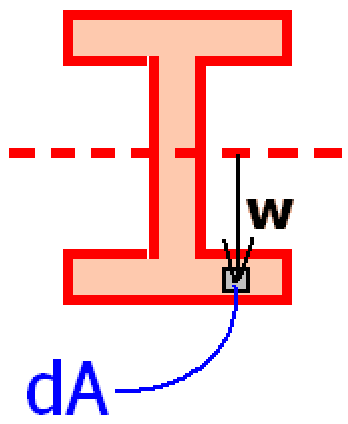
$$M+\partial M$$

$$= \iint_A w\,E\,\frac{w}{\rho}\,dA \;=\; EI/\rho$$

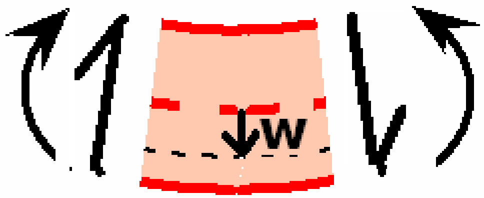
$$\Sigma F_{vert.} = (m/L)\,\partial x\,\frac{\partial^2 y}{\partial t^2} = -\partial V$$

Next, turning to the beam:

beam cross section



$$\iint_A w\,\sigma\,dA = M$$



$$M+\partial M$$

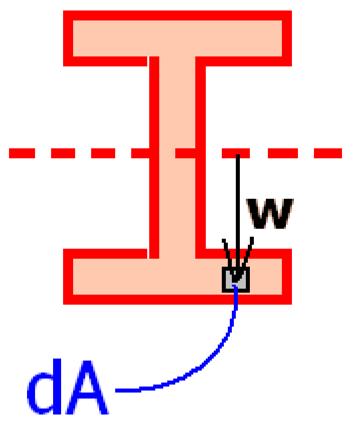
$$= \iint_A w\,E\,\frac{w}{\rho}\,dA = EI/\rho$$

$$\approx EI\,\frac{\partial^2 y}{\partial x^2}$$

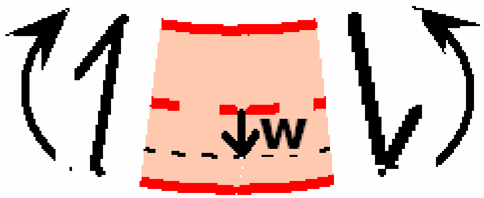
$$\Sigma F_{vert.} = (m/L)\,\partial x\,\frac{\partial^2 y}{\partial t^2} = -\partial V$$

But note that: $V = \frac{\partial M}{\partial x}$

beam cross section



$$\iint_A w\,\sigma\,dA = M$$



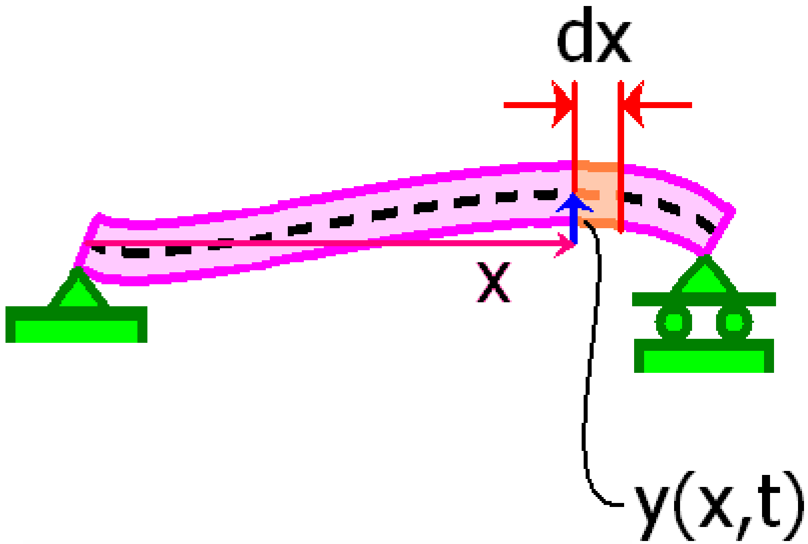
$$M+\partial M$$

$$= \iint_A w\,E\,\frac{w}{\rho}\,dA = EI/\rho$$

$$\approx EI\,\frac{\partial^2 y}{\partial x^2}$$

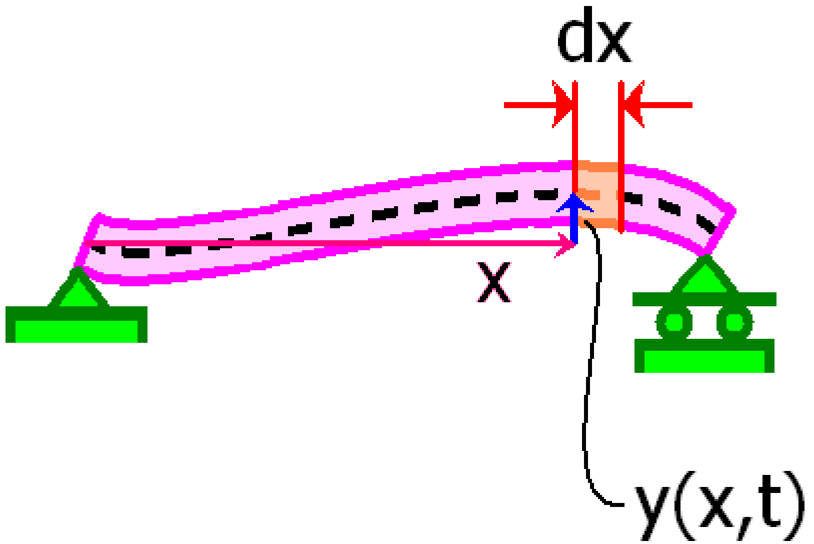
$$\Sigma F_{vert.} = (m/L)\,\partial x\,\frac{\partial^2 y}{\partial t^2} = -\partial V$$

Putting all of this together:



$$-(m/L)\partial x\frac{\partial^2 y}{\partial t^2} = \partial\frac{\partial M}{\partial x}$$

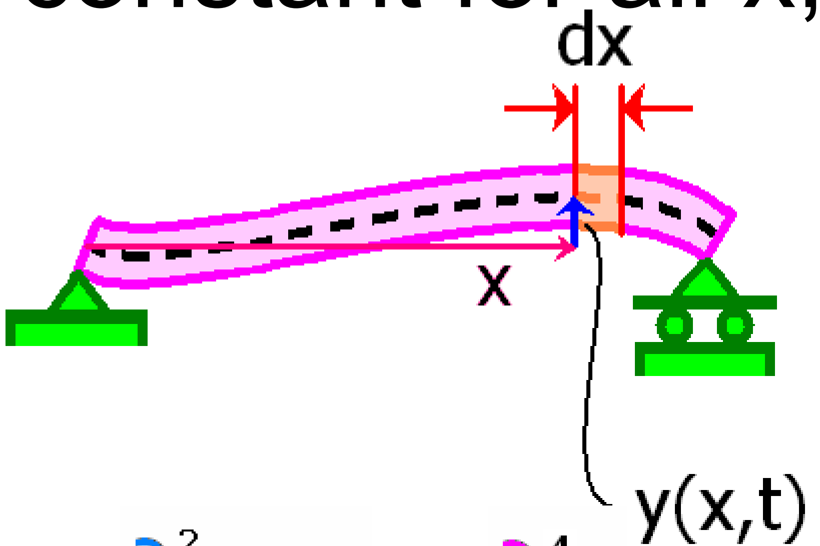
Putting all of this together:



$$-(m/L)\partial x \frac{\partial^2 y}{\partial t^2} = \partial \frac{\partial M}{\partial x}$$

$$-(m/L)\frac{\partial^2 y}{\partial t^2} \approx \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y}{\partial x^2}\right]$$

If the beam's cross section is the same across its length, such that EI is a constant for all x, then,

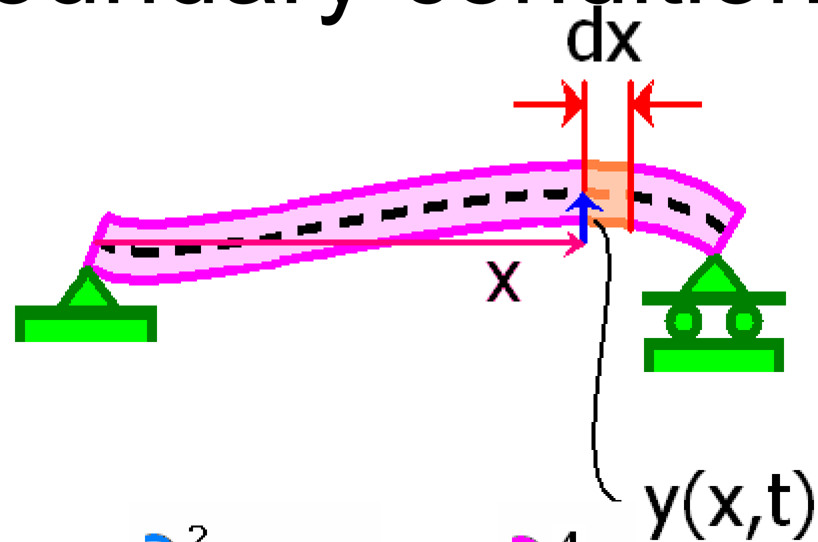


$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

where

$$a^2 = EIL/m$$

Because we have a fourth spatial derivative, we look for four boundary conditions



$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

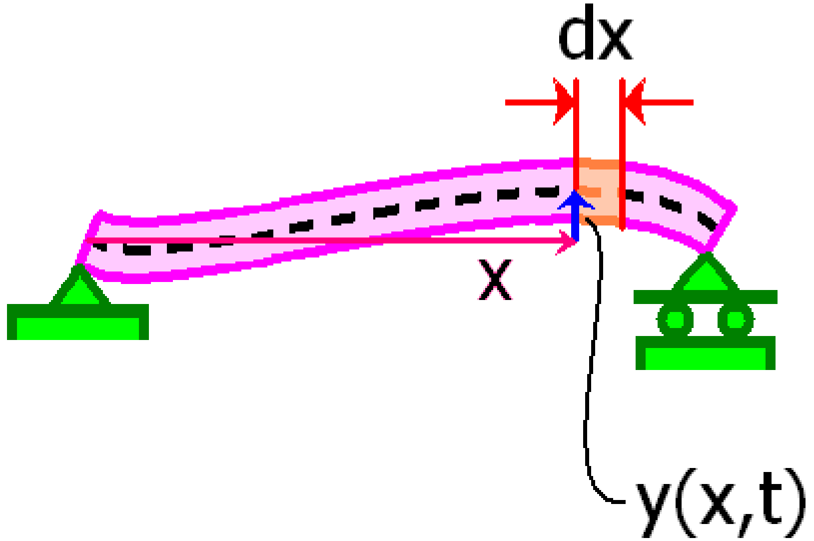
where

$$a^2 = EIL/m$$

Two of these are called “geometric boundary conditions”. They pertain to zero displacement at the two extreme ends:

$$y(0,t)=0$$

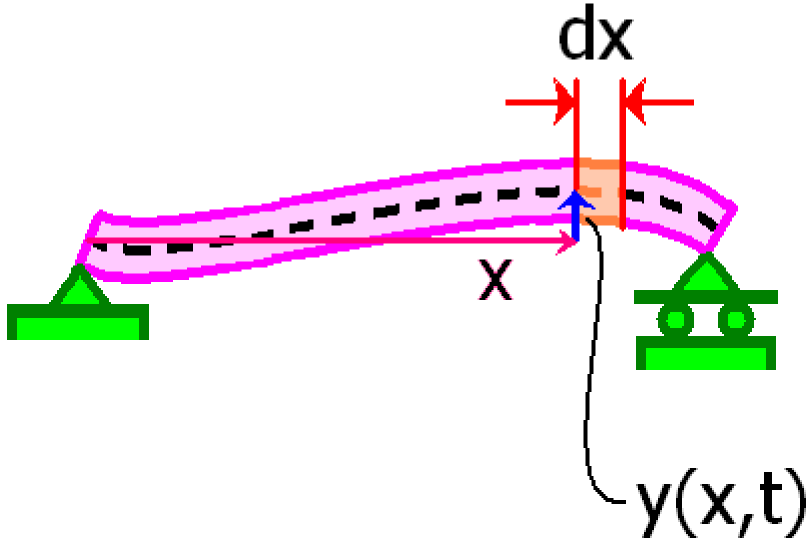
$$y(L,t)=0$$



The other two are identified with zero bending moment, considered “natural boundary conditions”, at each of the two ends. They may be written:

$$M(0,t)=EIy''(0,t)=0$$

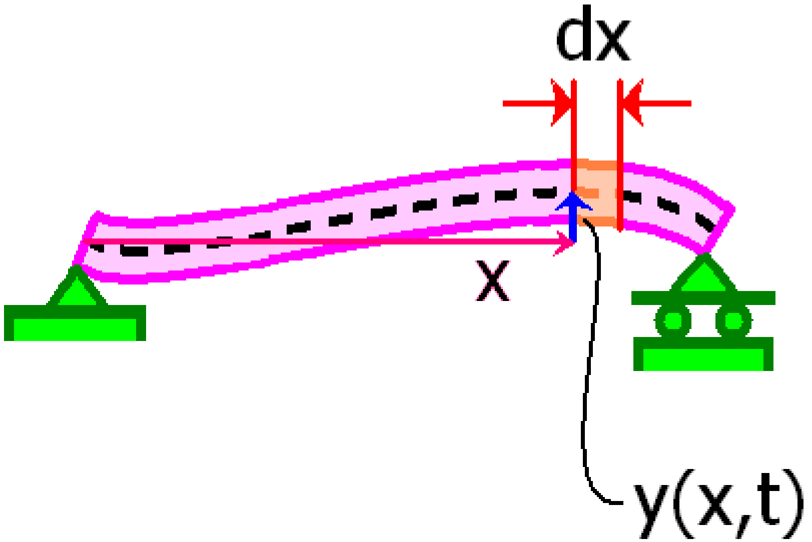
$$M(L,t)=EIy''(L,t)=0$$



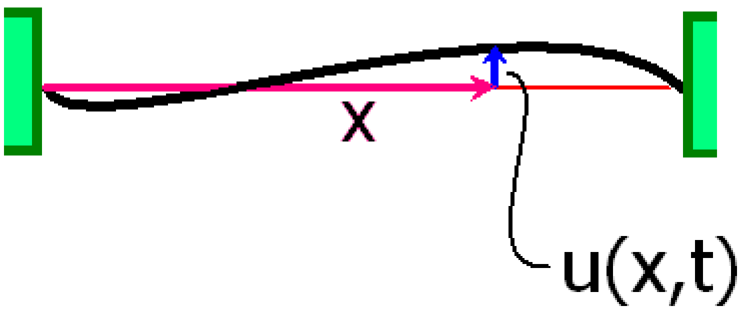
or:

$y''(0,t)=0$

$y''(L,t)=0$



Our two systems are governed according to:

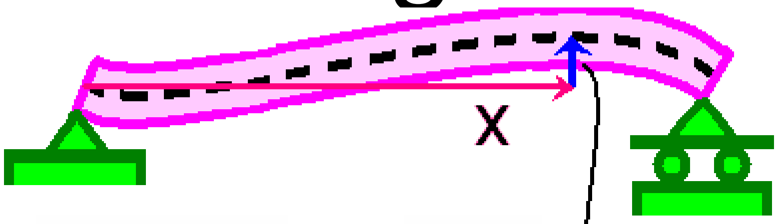


$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

where

$$a^2 = TL/m$$

$$u(0,t)=u(L,t)=0$$



$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

y(x,t)

where

$$a^2 = EIL/m$$

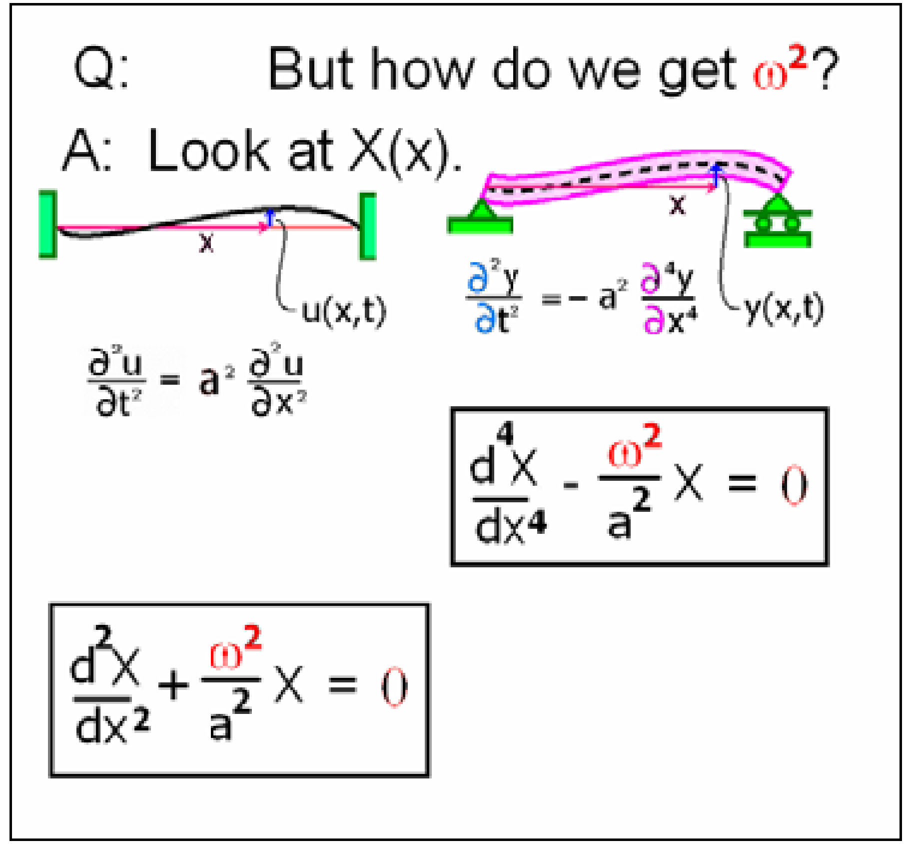
$$y(0,t)=0 \qquad y''(0,t)=0$$

$$y(L,t)=0 \qquad y''(L,t)=0$$

HW 37

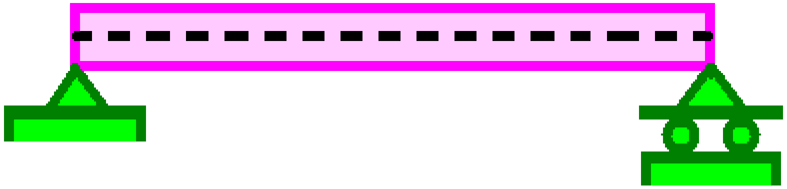
Due Dec. 5, 2005

Problem A: We consider the question of solving for the natural frequencies of the taut cable and the simply supported Euler-Bernoulli beam. Assuming for each of the two o.d.e.s on the right a solution form $X(x)=Ae^{rx}$, determine:



(a) A characteristic polynomial of second degree whose roots in r (denoted here as $r_1=i\omega/a$, $r_2=-i\omega/a$) admit a solution to the taut cable's (lower left of the two boxes) o.d.e.

(b) A characteristic polynomial of fourth degree whose 4 roots in r admit a solution to the simply supported beam's o.d.e.

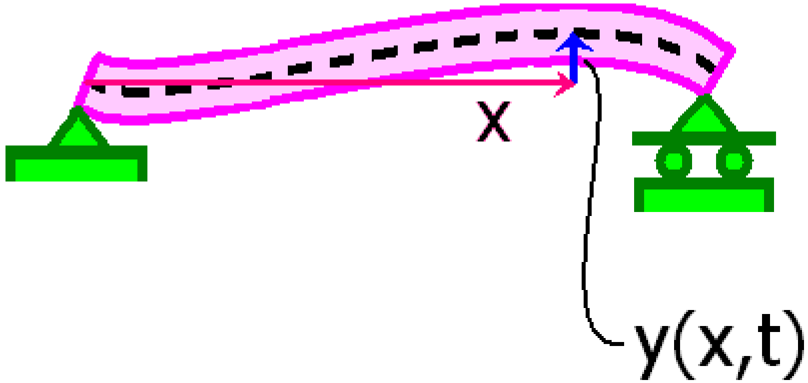


Problem B: Turning our attention to the beam: We may write the four roots of the characteristic equation: r1=**sqrt**(ω/a), r2=- **sqrt**(ω/a), r3=i **sqrt**(ω/a), r4=-i **sqrt**(ω/a). Show that the general solution:

X(x)
$= A_1 \exp(r_1 x) + A_2 \exp(r_2 x) + A_3 \exp(r3x) + A_4 \exp(r4x)$

may also be written

X(x) = C_1 cosh[**sqrt**(ω/a)x] + C_2 sinh[**sqrt**(ω/a)x] +
C_3 cos[**sqrt**(ω/a)x] + C_4 sin[**sqrt**(ω/a)x]



Let the dependent variable, y or u, be X(x)T(t)

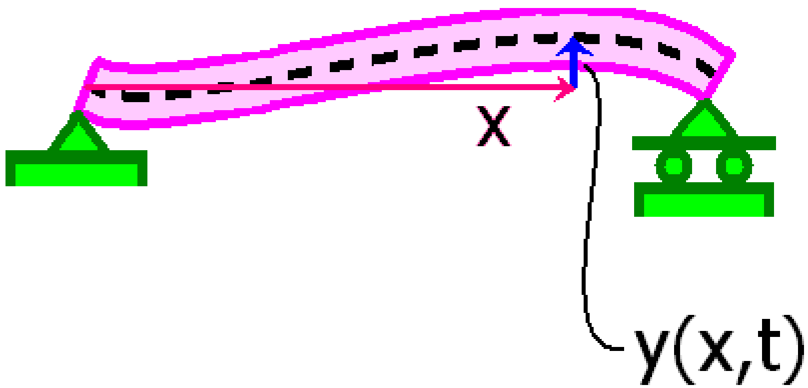
$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

where

$$a^2 = EIL/m$$

$y(0,t)=0$ $\quad$ $M(0,t)=EIy''(0,t)=0$

$y(L,t)=0$ $\quad$ $M(L,t)=EIy''(L,t)=0$



Let the dependent variable, y or u, be X(x)T(t)

$$X(x)\frac{d^2T}{dt^2} = -a^2\,T(t)\frac{d^4X}{dx^4}$$

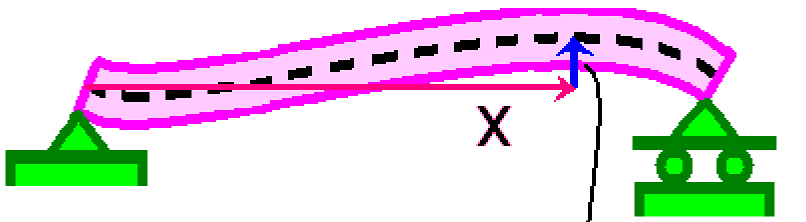
$$\frac{\partial^2 y}{\partial t^2} = -a^2\frac{\partial^4 y}{\partial x^4}$$

where

$$a^2 = EIL/m$$

$y(0,t)=0 \quad M(0,t)=EIy''(0,t)=0$

$y(L,t)=0 \quad M(L,t)=EIy''(L,t)=0$



Let the dependent variable, y or u, be X(x)T(t)

$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

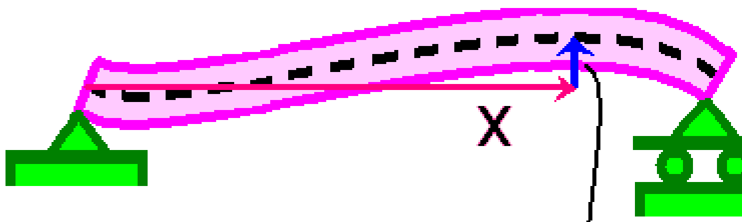
$$X(x)\frac{d^2T}{dt^2} = -a^2\, T(t)\frac{d^4X}{dx^4}$$

where

$$a^2 = EIL/m$$

$y(0,t)=0 \quad M(0,t)=EIy''(0,t)=0$

$y(L,t)=0 \quad M(L,t)=EIy''(L,t)=0$



Let the dependent variable, y or u, be X(x)T(t)

$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

$$X(x)\frac{d^2T}{dt^2} = -a^2 T(t)\frac{d^4X}{dx^4}$$

where

$$a^2 = EIL/m$$

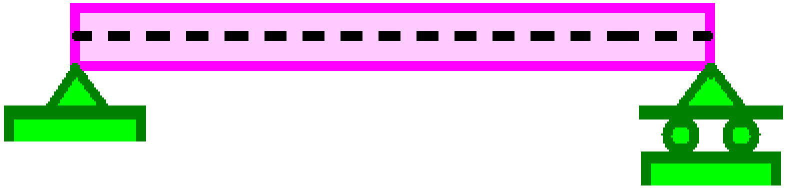
$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$y(0,t)=0 \qquad M(0,t)=EIy''(0,t)=0$

$y(L,t)=0 \qquad M(L,t)=EIy''(L,t)=0$

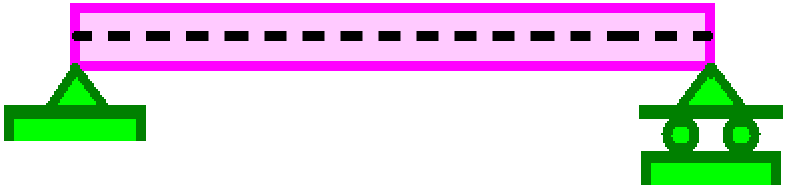
y(x,t) = X(x)T(t)

$X(0)=0$
$X(L)=0$
$X''(0)=0$
$X''(L)=0$



$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] + C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$$



$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

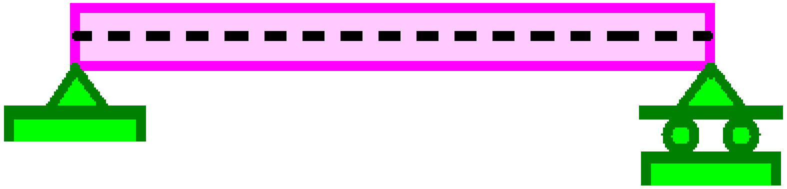
$X(0)=0$

$X(L)=0$

$X''(0)=0$

$X''(L)=0$

$$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] + C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$$

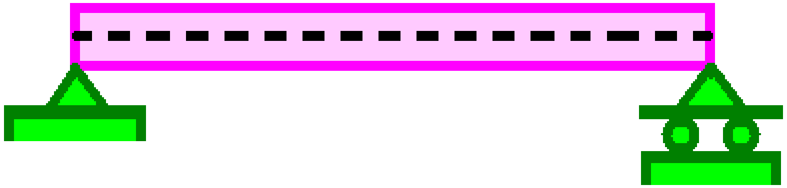


$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$X(0)=0$

$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] +$
$C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$

$C_1 + C_3 = 0$

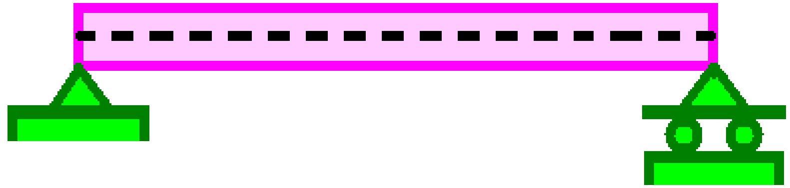


$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] +$
$C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$ $X''(0)=0$

$$C_1 + C_3 = 0$$

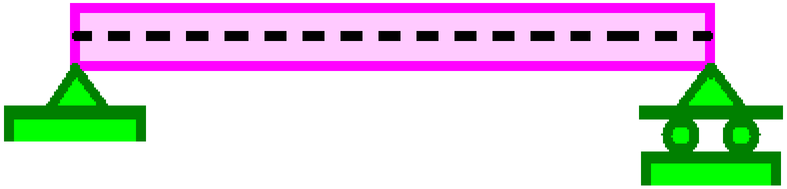
$$C_1 - C_3 = 0$$



$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] + C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x] \qquad X''(0)=0$$

$$\left.\begin{array}{l} C_1 + C_3 = 0 \\ C_1 - C_3 = 0 \end{array}\right\rangle \; C_1 = C_3 = 0$$

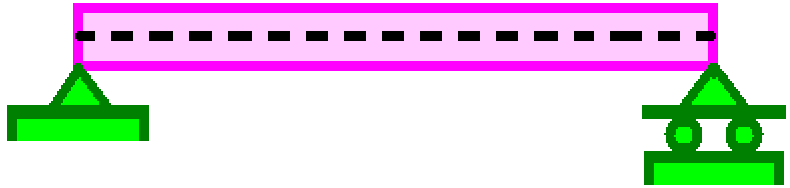


$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] +$
$C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$

$X(L)=0$

$$C_2 \sinh\sqrt{\omega/a}\,L + C_4 \sin\sqrt{\omega/a}\,L = 0$$



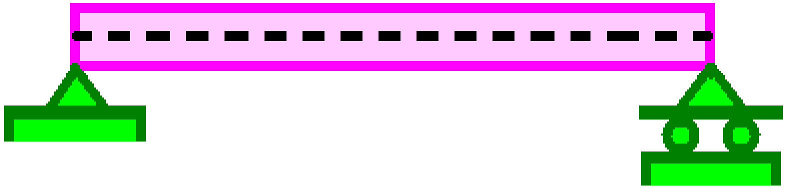
$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

$$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] + C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$$

$$X''(L)=0$$

$$C_2 \sinh\sqrt{\omega/a}\,L + C_4 \sin\sqrt{\omega/a}\,L = 0$$

$$[\omega/a][\,C_2 \sinh\sqrt{\omega/a}\,L - C_4 \sin\sqrt{\omega/a}\,L\,] = 0$$

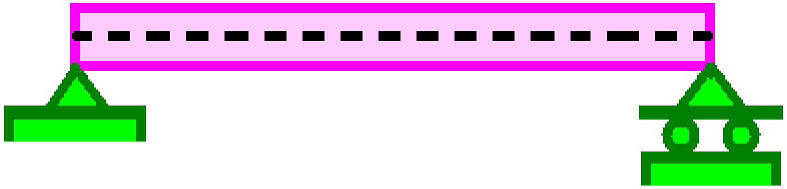


$$\boxed{\frac{d^4X}{dx^4} - \frac{\omega^2}{a^2}X = 0}$$

X(x) = C_1 cosh[**sqrt**(ω/a)x] + C_2 sinh[**sqrt**(ω/a)x] +
C_3 cos[**sqrt**(ω/a)x] + C_4 sin[**sqrt**(ω/a)x]

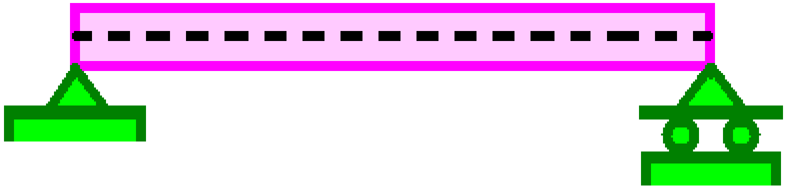
$$X''(L)=0$$

$$\begin{bmatrix} \sinh\sqrt{\omega/a}\,L & \sin\sqrt{\omega/a}\,L \\ \sinh\sqrt{\omega/a}\,L & -\sin\sqrt{\omega/a}\,L \end{bmatrix} \begin{Bmatrix} C_2 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$



To avoid the trivial solution, select ω such that:

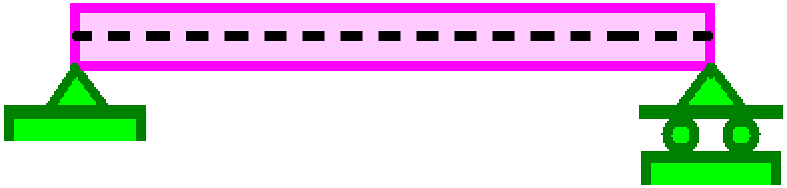
$$\begin{bmatrix} \sinh\sqrt{\omega/a}\,L & \sin\sqrt{\omega/a}\,L \\ \sinh\sqrt{\omega/a}\,L & -\sin\sqrt{\omega/a}\,L \end{bmatrix} \begin{Bmatrix} C_2 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$



To avoid the trivial solution,
select ω such that:

$$\begin{vmatrix} \sinh\sqrt{\omega/a}\,L & \sin\sqrt{\omega/a}\,L \\ \sinh\sqrt{\omega/a}\,L & -\sin\sqrt{\omega/a}\,L \end{vmatrix} = 0$$

$$\begin{bmatrix} \sinh\sqrt{\omega/a}\,L & \sin\sqrt{\omega/a}\,L \\ \sinh\sqrt{\omega/a}\,L & -\sin\sqrt{\omega/a}\,L \end{bmatrix} \begin{Bmatrix} C_2 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

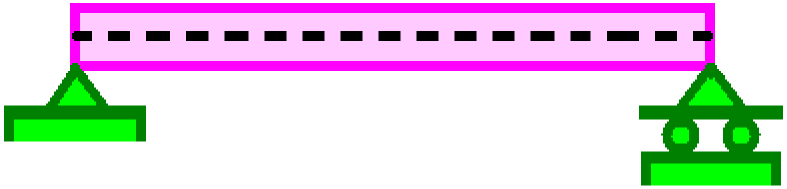


To avoid the trivial solution, select ω such that:

$$\begin{vmatrix} \sinh\sqrt{\omega/a}\,L & \sin\sqrt{\omega/a}\,L \\ \sinh\sqrt{\omega/a}\,L & -\sin\sqrt{\omega/a}\,L \end{vmatrix} = 0$$

$$2\ \sinh\sqrt{\omega/a}\,L\ \ \sin\sqrt{\omega/a}\,L = 0$$

For illustration take: a=1 L=1

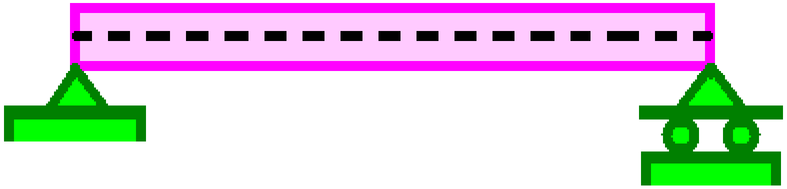


To avoid the trivial solution, select ω such that:

$$\begin{vmatrix} \sinh\sqrt{\omega/a}\,L & \sin\sqrt{\omega/a}\,L \\ \sinh\sqrt{\omega/a}\,L & -\sin\sqrt{\omega/a}\,L \end{vmatrix} = 0$$

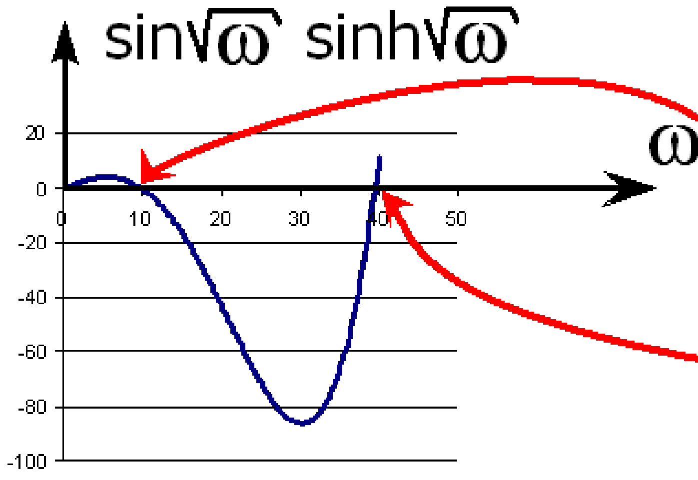
$$2\,\sinh\sqrt{\omega/a}\,L \quad \sin\sqrt{\omega/a}\,L = 0$$

For illustration take: a=1 L=1



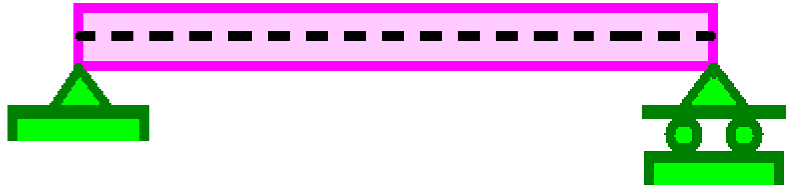
To avoid the trivial solution, select ω such that:

$$2\,\sinh\sqrt{\omega/a}\,L\quad \sin\sqrt{\omega/a}\,L = 0$$



$\omega_1 = 9.870\ s^{-1}$
$\omega_2 = 39.478\ s^{-1}$

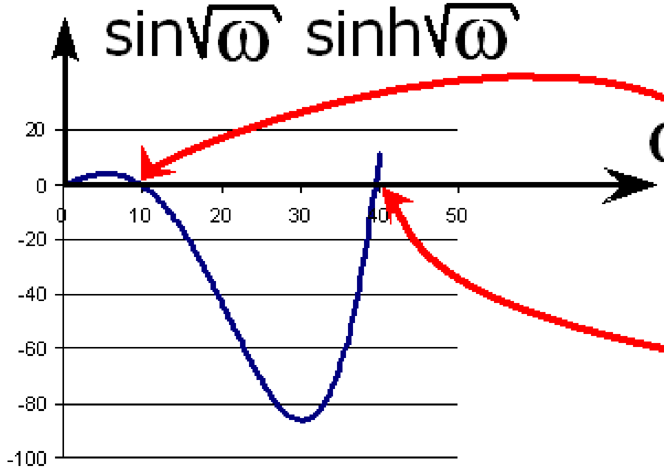
For illustration take: $a=1$ $L=1$



Either of the previous equations can be applied to relate C_2 and C_4:

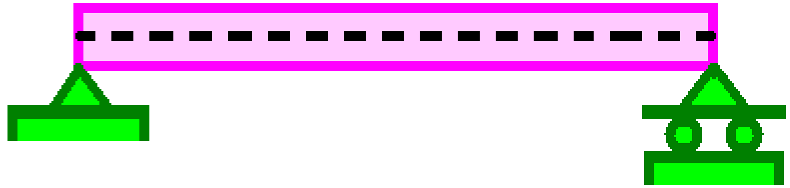
$$C_2 \sinh\sqrt{\omega/a}\,L + C_4 \sin\sqrt{\omega/a}\,L = 0$$

$$[\omega/a][C_2 \sinh\sqrt{\omega/a}\,L - C_4 \sin\sqrt{\omega/a}\,L] = 0$$



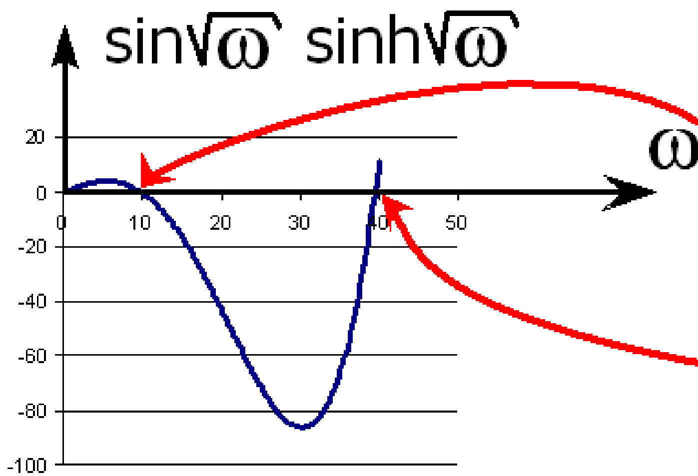
$\omega_1 = 9.870\ s^{-1}$
$\omega_2 = 39.478\ s^{-1}$

For illustration take: a=1 L=1



Let's use the first of the two with the first natural frequency:

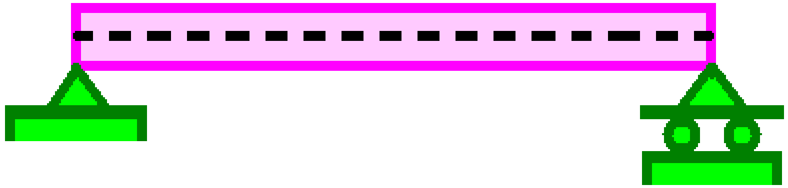
$$C_2 \sinh\sqrt{9.870} + C_4 \sin\sqrt{9.870} = 0$$



$\omega_1 = 9.870\ s^{-1}$

$\omega_2 = 39.478\ s^{-1}$

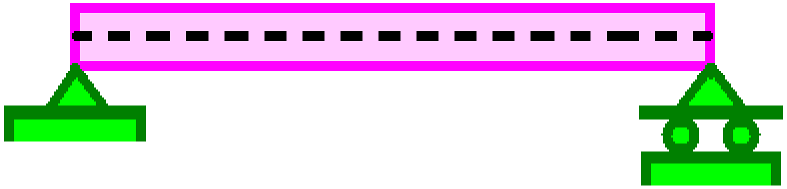
For illustration take: a=1 L=1



Let's use the first of the two with the first natural frequency:

$$C_2 \sinh\sqrt{9.870} + \cancelto{0}{C_4 \sin\sqrt{9.870}} = 0$$

For illustration take: a=1 L=1

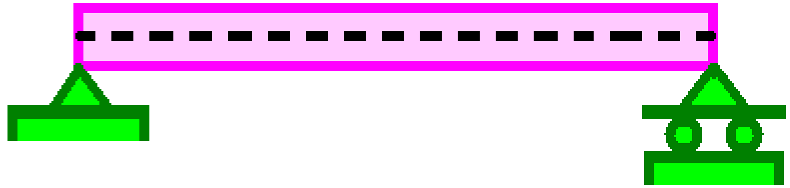


Let's use the first of the two with the first natural frequency:

$$C_2 \sinh\sqrt{9.870} + \overbrace{C_4 \sin\sqrt{9.870}}^{0} = 0$$

$$\therefore C_2 = 0$$

For illustration take: a=1 L=1



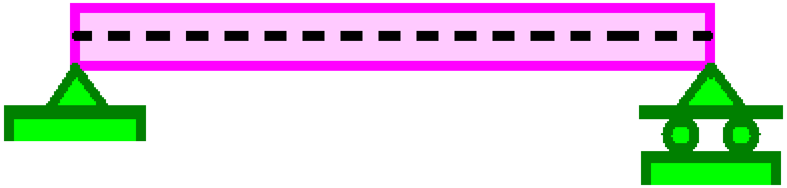
Let’s use the first of the two with the first natural frequency:

$$C_2 \sinh\sqrt{9.870} + C_4 \overbrace{\sin\sqrt{9.870}}^{0} = 0$$

$$\therefore\ C_2 = 0$$

$$X(x) = C_1 \cosh[\mathbf{sqrt}(\omega/a)x] + C_2 \sinh[\mathbf{sqrt}(\omega/a)x] +$$
$$C_3 \cos[\mathbf{sqrt}(\omega/a)x] + C_4 \sin[\mathbf{sqrt}(\omega/a)x]$$

For illustration take: a=1 L=1



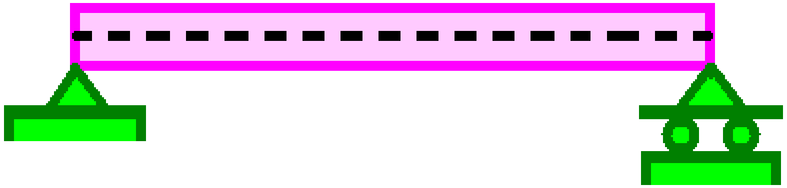
Let's use the first of the two with the first natural frequency:

$$C_2 \sinh\sqrt{9.870} + C_4 \cancelto{0}{\sin\sqrt{9.870}} = 0$$

$$\therefore \; C_2 = 0$$

$$X_1(x) = C_4 \sin\sqrt{9.870}\, x$$

For illustration take: a=1 L=1



Let's use the first of the two with the first natural frequency:

recall: $y(x,t)=X(x)T(t)$

$T(t)=c_1 \cos \omega_1 t + d_1 \sin\omega_1 t$

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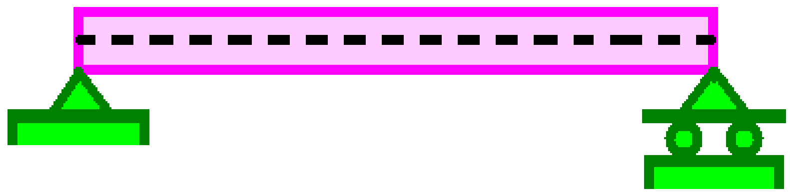
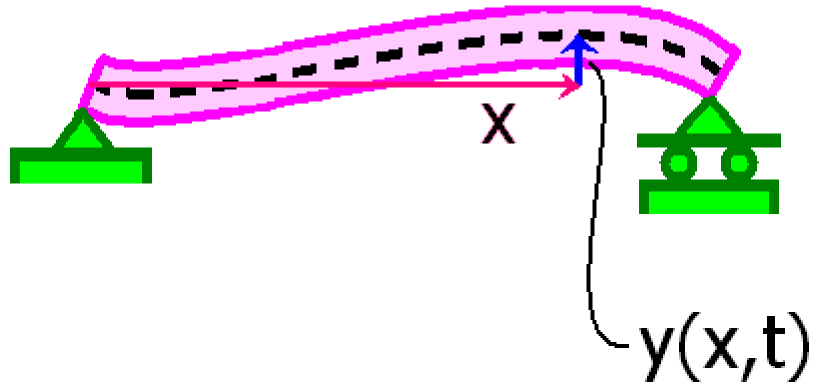


Let's use the first of the two with the first natural frequency:

recall: $y(x,t)=X(x)T(t)$

$$T(t)=c_1 \cos \omega_1 t + d_1 \sin \omega_1 t$$

$$X_1(x) = \cancelto{1}{C_4} \sin\sqrt{9.870}\, x$$



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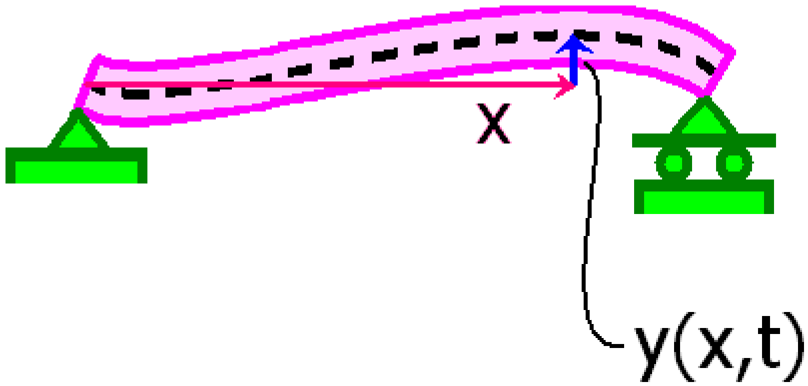
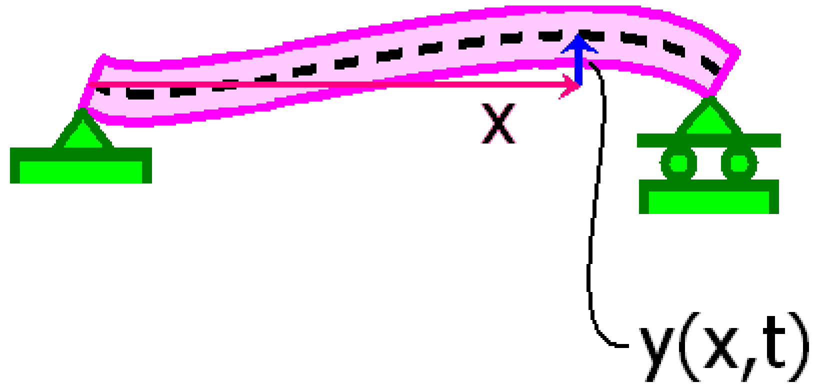


recall: $y(x,t) = \sum X_i(x) T_i(t)$

$T_i(t) = c_i \cos \omega_i t + d_i \sin \omega_i t$



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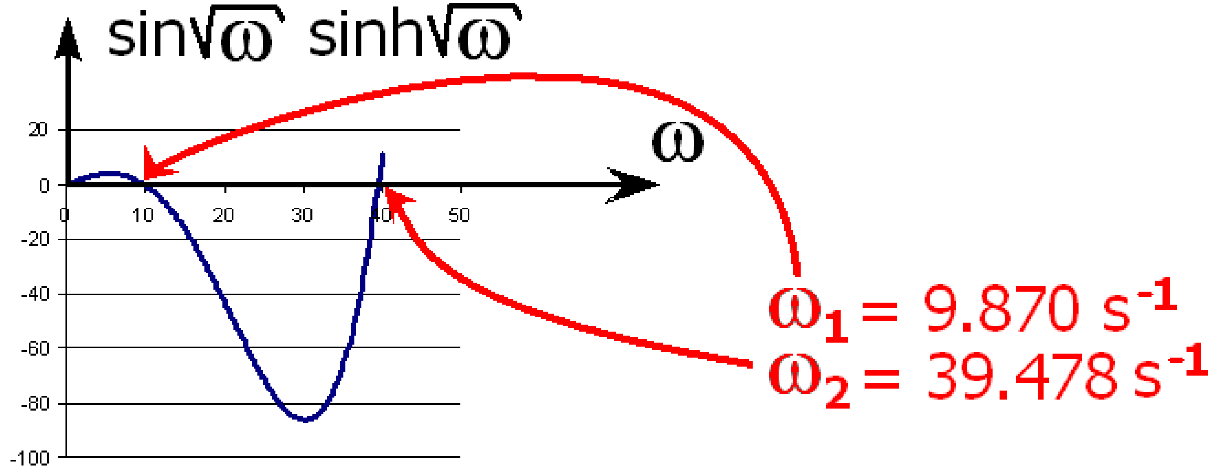
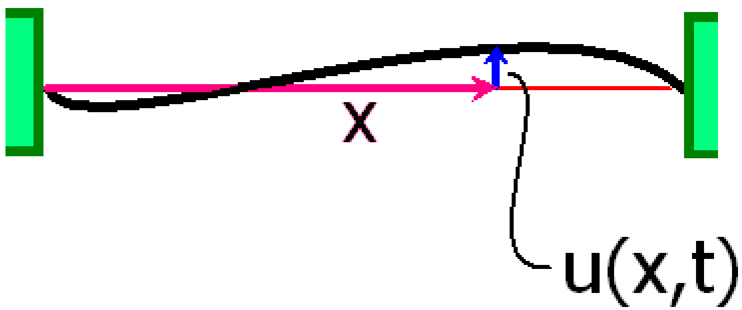


The simply supported beam turns out to have the same mode shapes as the taut cable.



For a beam having a different type of support, this is not the case.

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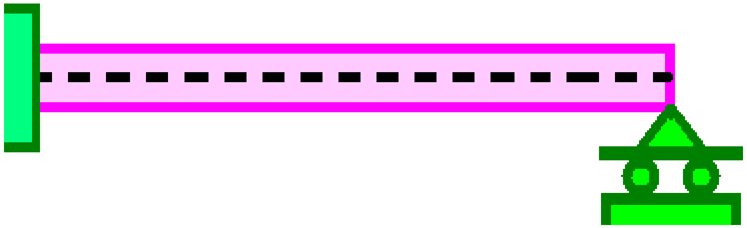
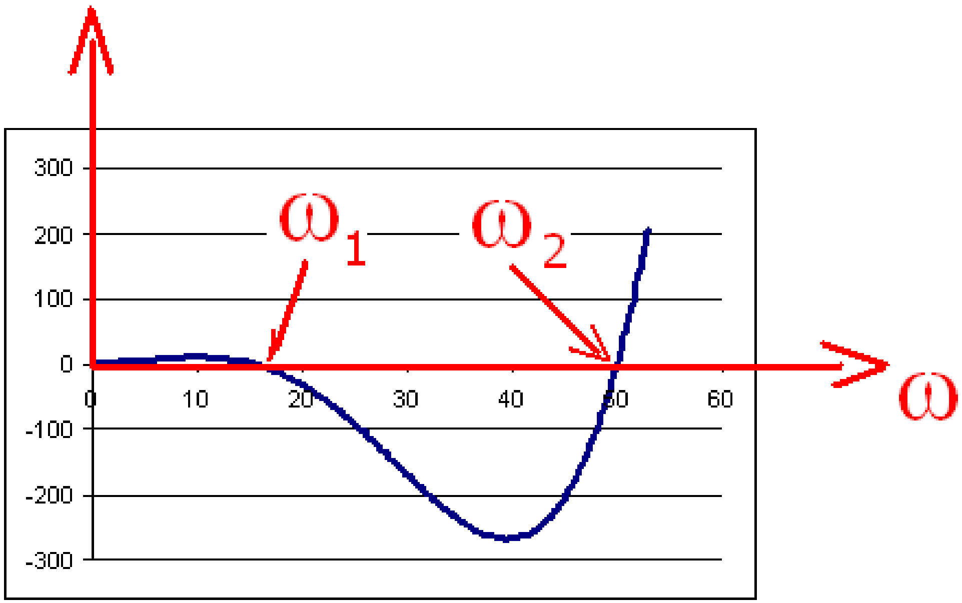
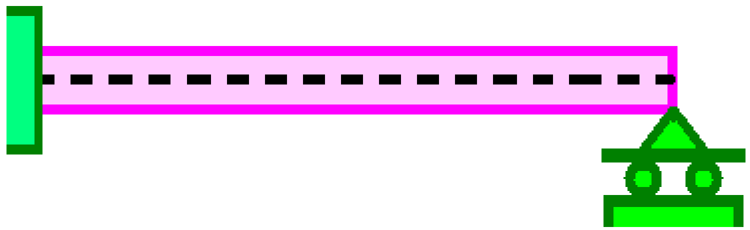


Last homework



Cantilever beam w/ pin support at right end: X(0)=0; X'(0)=0; X(L)=0; X"(L)=0.

Fourth natural frequency is 104.25 s^{-1}



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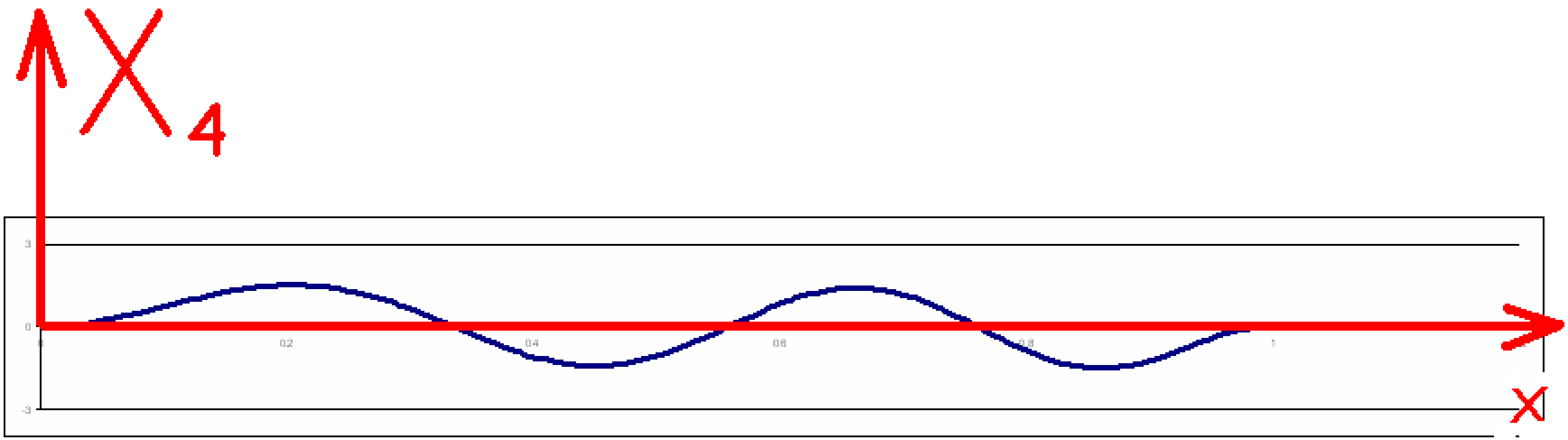
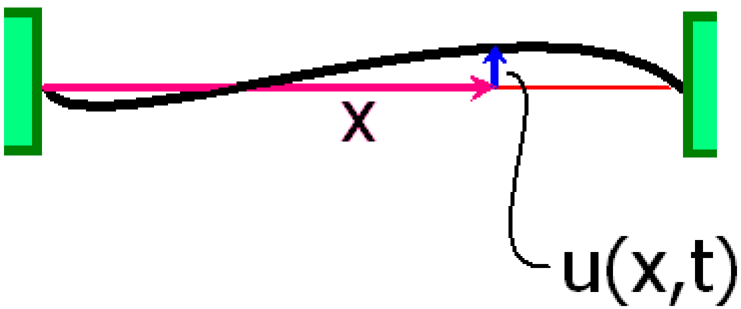


Our two systems are governed according to:

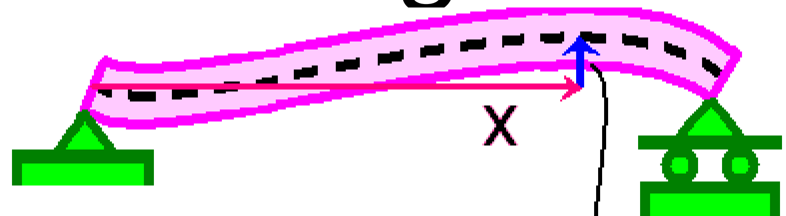


$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

where

$$a^2 = TL/m$$

$$u(0,t)=u(L,t)=0$$



$$\frac{\partial^2 y}{\partial t^2} = -a^2 \frac{\partial^4 y}{\partial x^4}$$

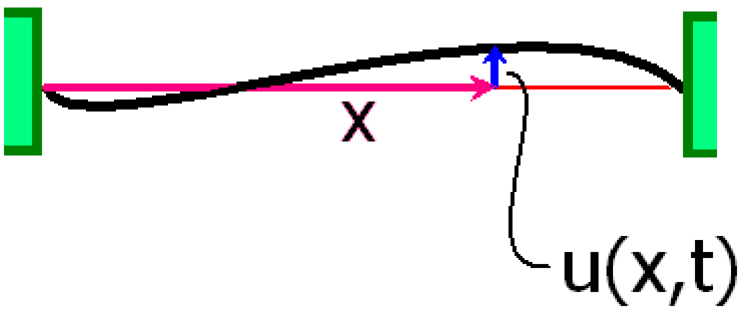
where

$$a^2 = EIL/m$$

$$y(0,t)=0 \qquad y''(0,t)=0$$

$$y(L,t)=0 \qquad y''(L,t)=0$$

Consider the vibrating cable on the left



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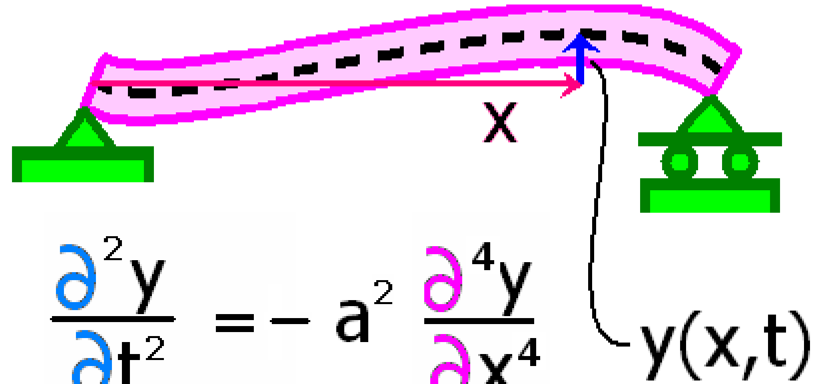

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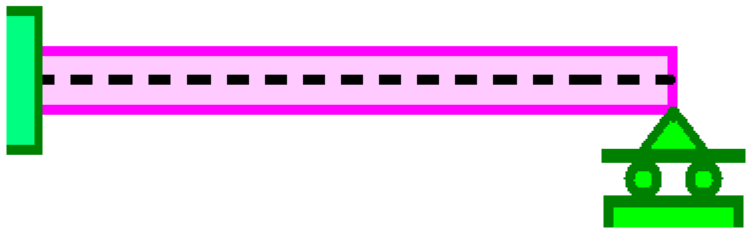
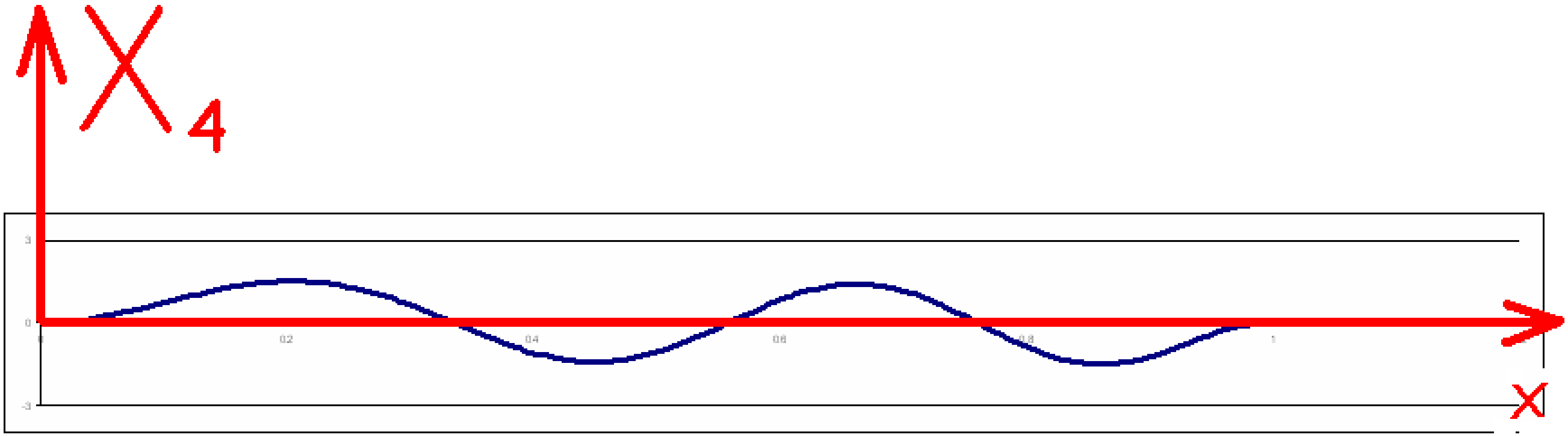
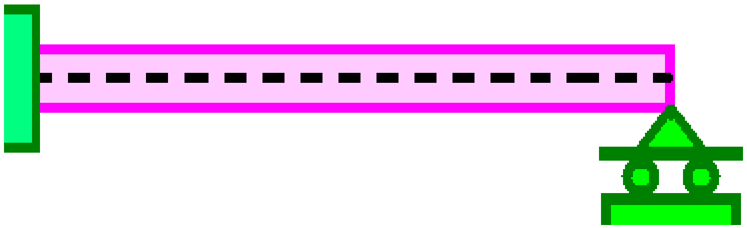
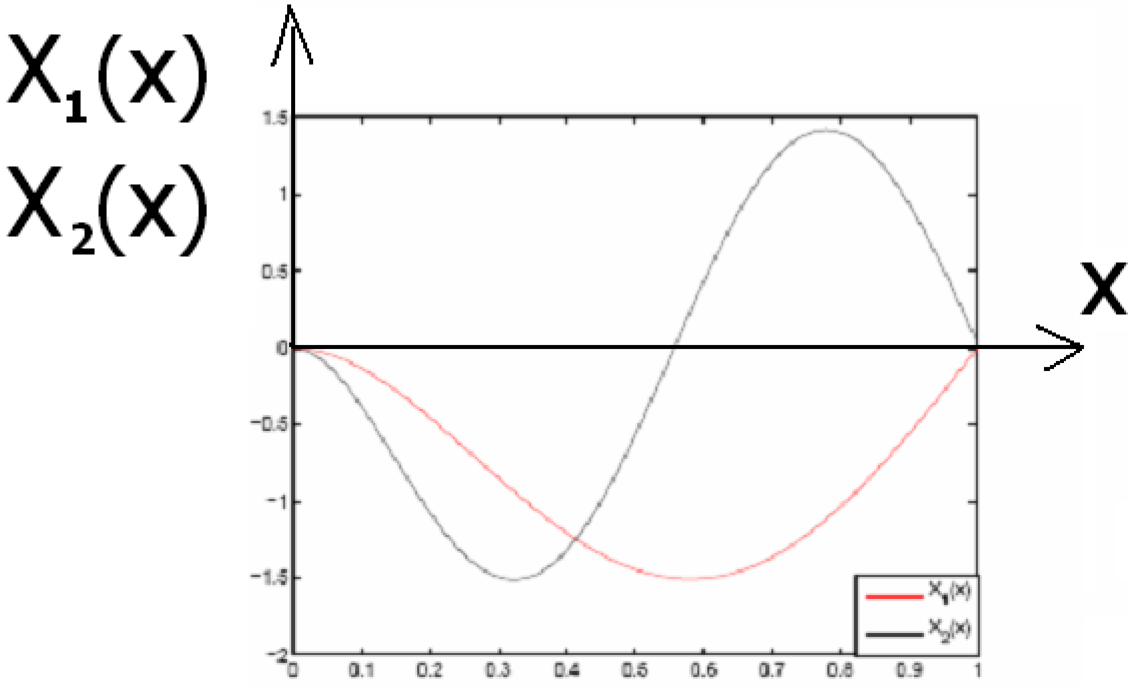


Cantilever beam w/ pin support at right end: $X(0)=0$; $X'(0)=0$; $X(L)=0$; $X''(L)=0$.



First two mode shapes





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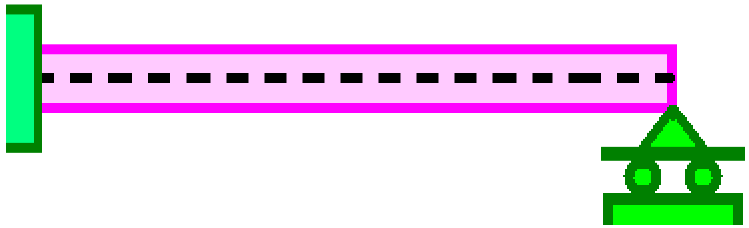

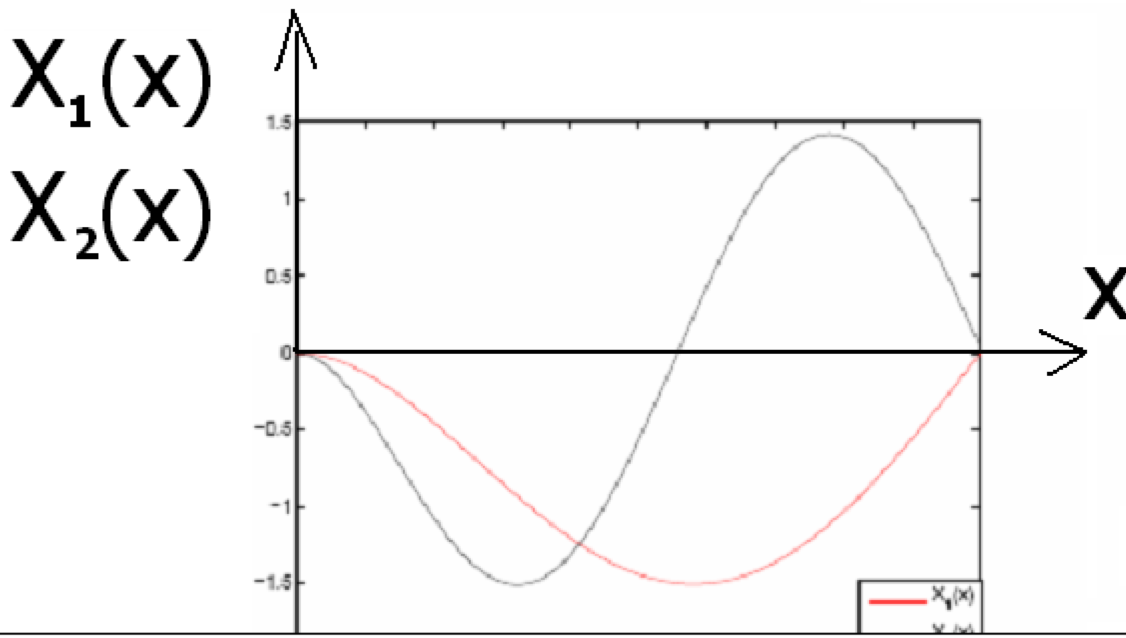

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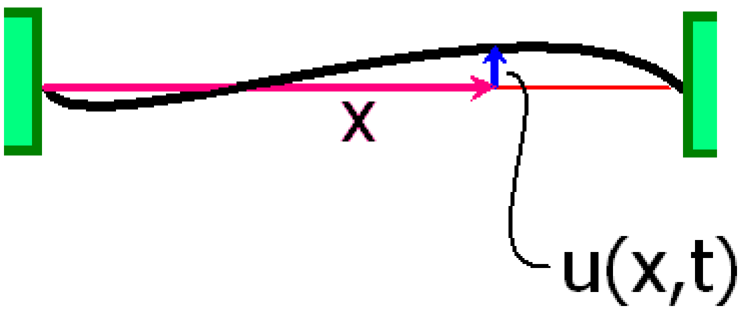
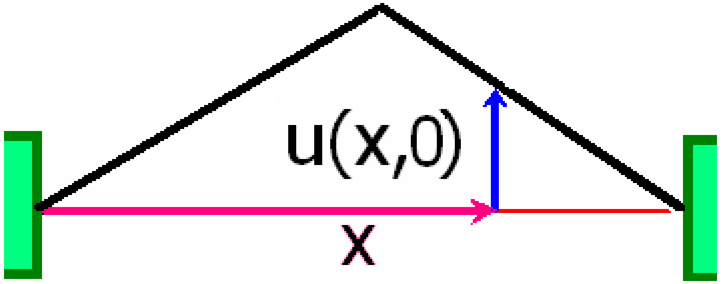


Initial conditions are comprised of a given u(x,0) and an assumption of zero initial velocity, i.e. $u_t(x,0)=0$.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$



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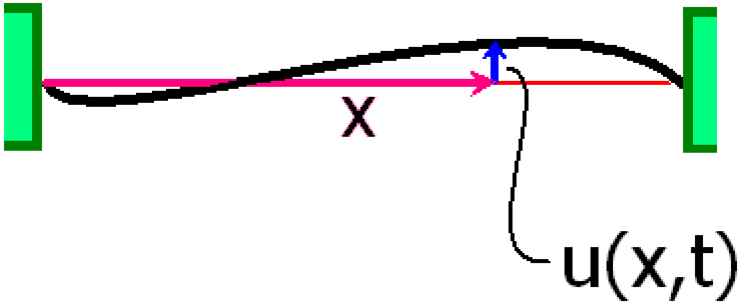

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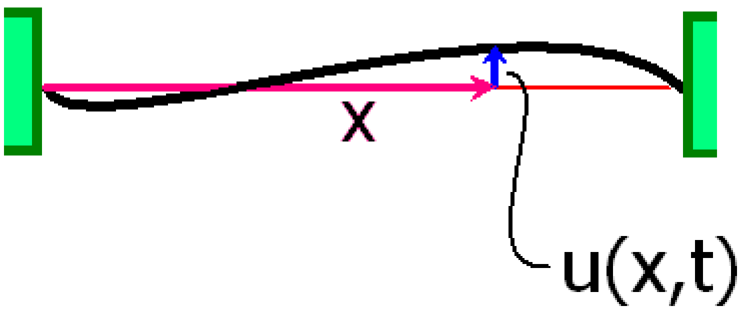

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We assumed a solution:

$u(x,t)=X(x)T(t)$

$T(t)=c\cos\omega_n t + d\sin\omega_n t$

$X(x) = \sin n\pi x/L$

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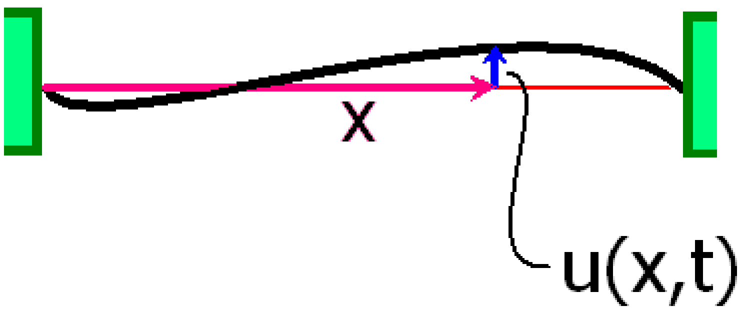

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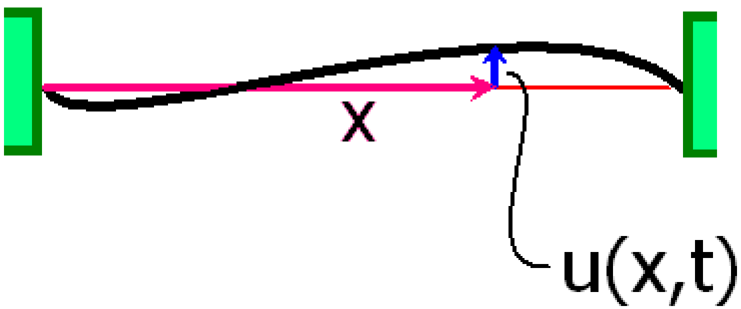

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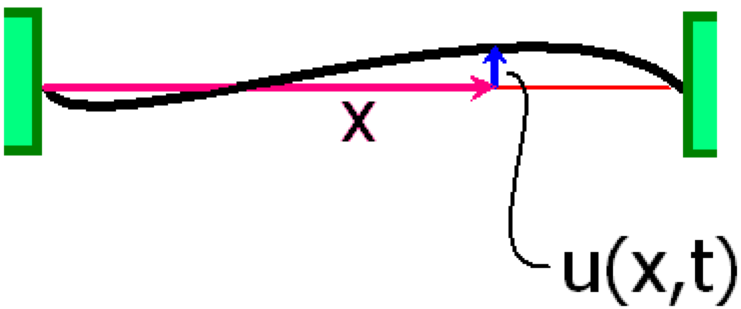

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$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Due to linearity, superposition:

$$u(x,t)= \sum_n X_n(x)T_n(t)$$

$$T_n(t)=c_n\cos\omega_n t + d_n\sin\omega_n t$$

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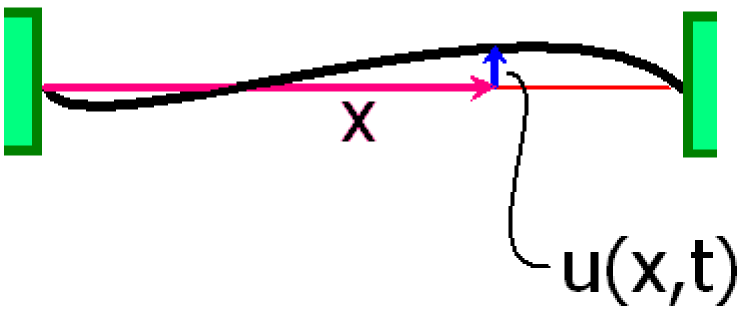
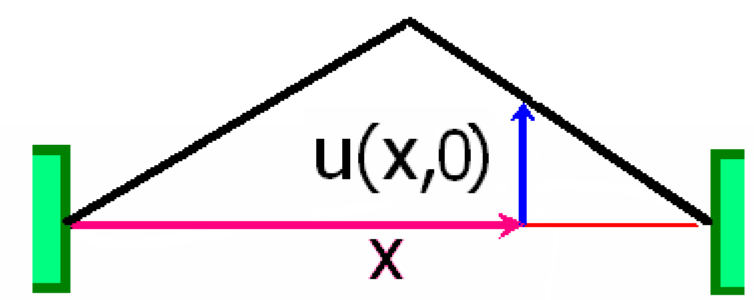


Think of our init. cond'n: u(x,0)

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$



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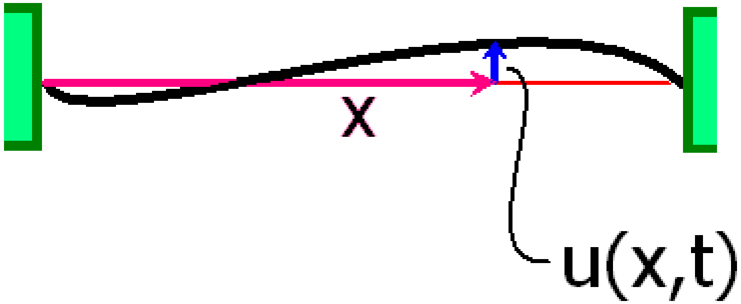
$n = 1, 2, 3 \ldots$

$$u(x,0) = \sum_n c_n \sin n\pi x/L$$

$$= \sum_n c_n X_n(x)$$



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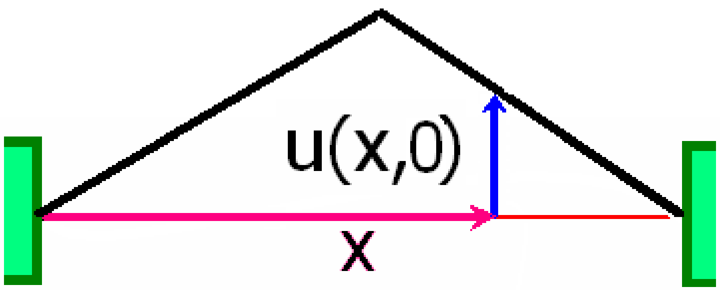

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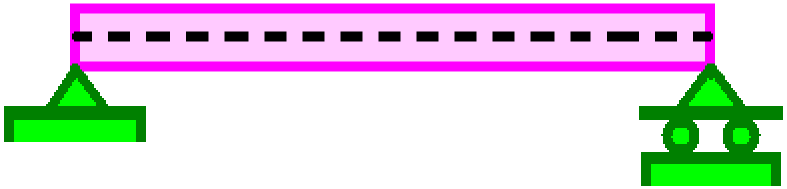
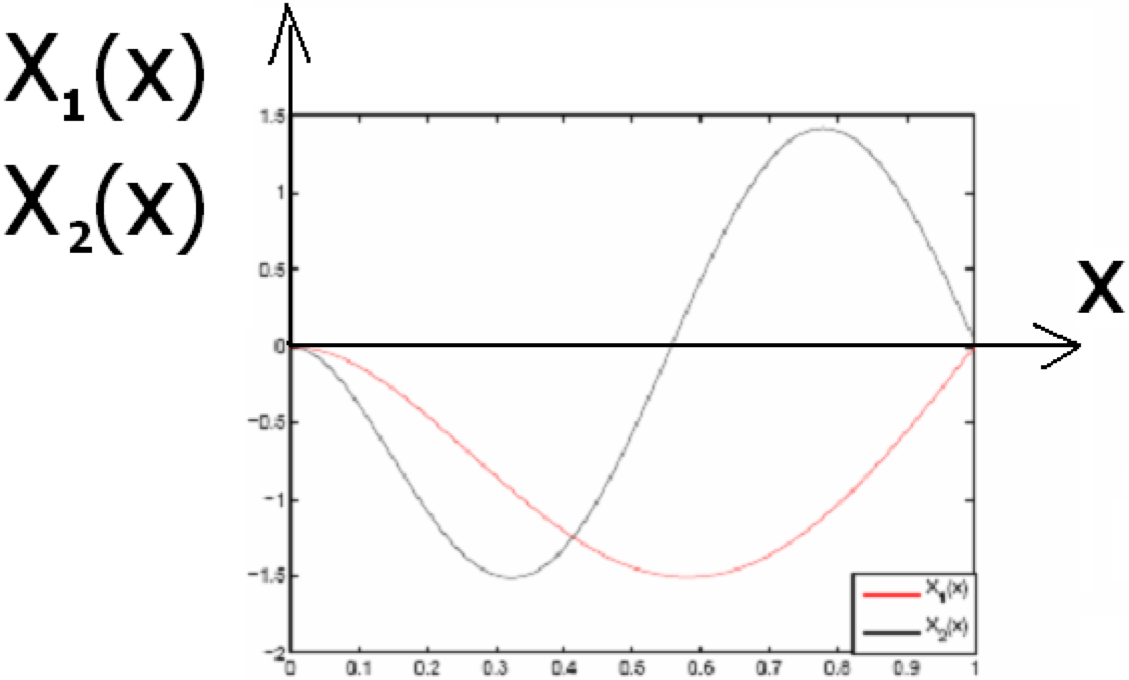
$$u(x,0) = \sum_{n} c_n X_n(x)$$

$$y(x,0) = \sum_n c_n X_n(x)$$





Final Monday, December 12, 2005, Room: 141 DeBartolo, Time: 1:45-3:45 PM.

8 problems

Please bring:

1. 3 cheat sheets
2. calculator
3. pencils
4. scratch paper

It is formatted like the midterms: eight problems total; 15 minutes/problem.