that the becquerel is much better for communicating among physicists (especially because it is unusual for those of us who are not nuclear or health physicists to be able to identify the amount of activity in the curie, that is, the activity in one gram of radium). I suggest future use of statements such as "a 370 000 Bq (equivalent to 10 μCi) source" to minimize reader confusion.

Reference 1 describes a nice experiment to determine that the relativistic relations $\mathbf{p}=\gamma m\mathbf{v}$ and $E=\gamma mc^2$ correctly characterize particles with large β. I wish it had been labeled that way.

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The longitudinal momentum of transverse traveling waves on a string

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Several years ago in this journal, Rowland and Pask[1] took note of a confusion in the literature regarding the longitudinal momentum carried by transverse traveling waves on a string. Through a combined numerical and analytical approach, they reached (what I take to be) a convincing resolution of the matter. Rowland and Pask accurately diagnosed the source of error by several authors in an unfounded assumption that the instantaneous velocity of an infinitesimal segment of string is always perpendicular to the segment. This is not the source of error, however, in the influential text by Elmore and Heald,[2] which is invariably cited by papers on the subject and contains a frequently quoted incorrect expression for momentum density. The purpose of this note is to identify the mistake, which is found in Chapter 1, Section 11, pages 46–47 of the book, as well as to demonstrate that by correcting this mistake we arrive at the result of Rowland and Pask.

In the notation of Elmore and Heald, the wave equation is

$$\frac{\partial^2\eta}{\partial t^2}=c^2\frac{\partial^2\eta}{\partial x^2}, \tag{1}$$

where $\eta(x,t)$ is the transverse displacement and $c=(\tau_0/\lambda_0)^{1/2}$ is the propagation velocity, with τ_0 the tension and λ_0 the linear mass density. Their treatment of longitudinal momentum starts with an expression for the longitudinal component of force density due to string curvature:

$$-\tau_0\frac{\partial^2\eta}{\partial x^2}\frac{\partial\eta}{\partial x}=\lambda_0\frac{\partial^2\xi}{\partial t^2} \tag{2}$$

with $\xi(x,t)$ the longitudinal displacement of the string resulting from the transverse wave.[3] Then, in their own words, "we integrate with respect to time from t_0, a time when no wave is present on the string, to an arbitrary later time t and with respect to x over a finite string segment lying between x_1 and x_2. The result should be the momentum G_x acquired by the string segment as the result of transverse wave motion." These integrations lead to the following:

$$G_x=-\lambda_0\int_{x_1}^{x_2}\frac{\partial\eta}{\partial x}\frac{\partial\eta}{\partial t}dx+\int_{t_0}^{t}[K_1(x_2,t)-K_1(x_1,t)]dt, \tag{3}$$

where $K_1=dK/dx=\frac{1}{2}\lambda_0(d\eta/dt)^2$ is the kinetic energy density due to transverse motion of the string.

According to Elmore and Heald, "[Eq. (3)] for the momentum G_x has the following interpretation: the second integral on the right clearly represents momentum delivered to the string segment by impulses at the two boundaries at x_1 and x_2. If these boundaries are very remote, so that a wave disturbance initiated on the string segment has not yet had time to reach them, this integral vanishes. We are thus left with the first integral, whose form suggests that the quantity

$$g_x(x,t)\equiv-\lambda_0\frac{\partial\eta}{\partial t}\frac{\partial\eta}{\partial x} \tag{4}$$

may be interpreted as a localized momentum density in the x direction associated with a transverse wave."

Here is the problem: the initiation of wave motion on a previously quiescent segment of string requires either that a wave propagates onto that segment from elsewhere on the string, or that external forces—other than the forces associated with wave propagation itself—are imposed from outside the system. The first alternative directly violates the authors' assumption that both ends of the string remain undisturbed during the integration time. The second alternative admits forces not described by the expression being integrated.

Let us adopt the first alternative, allowing for the wave to propagate onto the string segment at x_1. Then we have

$$\begin{aligned}\int_{t_0}^{t}K_1(x_1,t)dt&=\frac{1}{2}\lambda_0\int_{t_0}^{t}\left(\frac{\partial\eta}{\partial t}\right)^2dt\\&=\frac{1}{2}\lambda_0\int_{t_0}^{t}\frac{\partial\eta}{\partial t}\frac{\partial\eta}{\partial t}dt\\&=-\frac{1}{2}\lambda_0\int_{x_1}^{x_2}\frac{\partial\eta}{\partial t}\frac{\partial\eta}{\partial x}dx,\end{aligned} \tag{5}$$

the last equality holding by virtue of $(\partial y/\partial t)dt = -(\partial y/\partial x)dx$ for the traveling wave, which permits us to convert the time integral to an integral over x at time t (whereas the time integrals were carried out at $x=x_1$). This integral, inserted into Eq. (3), just cancels half the first integral on the right-hand side of the equation, leaving the result

$$G_x = -\frac{1}{2}\lambda_0 \int_{x_1}^{x_2} \frac{\partial \eta}{\partial t}\frac{\partial \eta}{\partial x}\,dx, \tag{6}$$

which leads to identification of

$$g_x = -\frac{1}{2}\lambda_0 \frac{\partial \eta}{\partial t}\frac{\partial \eta}{\partial x} \tag{7}$$

as the momentum density. This is the result obtained by Rowland and Pask, under the usual conditions pertaining to transverse waves on a string.

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[3]We are speaking here of small longitudinal motions associated with a transverse wave. The wave equation itself is derived under the approximation of purely transverse motion and uniform tension. Due to curvature of the string, the tension forces at opposite ends of an infinitesimal segment do not cancel. The longitudinal component of the resulting net force is much smaller than the transverse component if we have $|\partial\eta/\partial x| \ll 1$. The longitudinal motions may then be treated as a perturbation on the dominant transverse motions. Longitudinal motions are also produced by the variations in string tension associated with *longitudinal* waves; indeed, the physical impetus which establishes a transverse wave is likely to generate a longitudinal wave too, and longitudinal waves are essential to the conservation of momentum when a transverse wave encounters a density discontinuity. Interested readers should consult Rowland and Pask.

Impedance between adjacent nodes of infinite uniform *D*-dimensional resistive lattices

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Infinite resistive network problems have served as excellent vehicles for helping electrical engineering and physics students recognize and appreciate the power of superposition and symmetry in the analysis of electrical networks. These problems have been studied extensively using superposition and symmetry.[1–10] A special case of this class of problems involves the calculation of the effective resistance between two adjacent nodes of an infinite uniform two-dimensional (2D) resistive lattice (periodic in both dimensions with a zero-potential boundary condition at infinity) comprised of identical resistors each of value R. In particular, the effective resistance between two adjacent nodes of the 2D Liebman resistive mesh (the infinite 2D square resistive lattice) was calculated by Aitchison[1] and found to be $(1/2)R$. Bartis[2] calculated the resistance between adjacent nodes for three other infinite 2D resistive lattices, the triangular, Honeycomb, and Kagomé lattices, and found the effective resistances to be $(1/3)R$, $(2/3)R$, and $(1/2)R$, respectively.

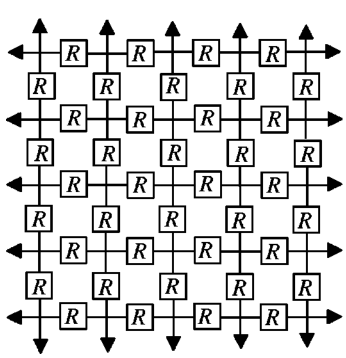

Fig. 1. Infinite 2D square resistive lattice.

The goal of this paper is to extend the results of Refs. 1 and 2 to the general problem of finding the total effective resistance R_{eff} between two adjacent nodes of any infinite D-dimensional resistive lattice, where $D=1, 2, 3, \ldots$ and the lattice is periodic and infinite in all D dimensions with a zero-potential boundary condition at infinity. Our general solution for R_{eff} is of pedagogical interest because it generalizes the previous results of Refs. 1 and 2 to a simple and elegant equation that covers all adjacent-node infinite D-dimensional resistive networks and because it reinforces the power of the superposition principle and symmetry in electrical circuit analysis.

For the purpose of illustration, consider the infinite 2D square resistive lattice shown in Fig. 1. The number of resistors connected to each node is denoted by M ($M=4$ in Fig. 1). As in Refs. 1 and 2, we use superposition and symmetry along with two test current sources each of value I to calculate the effective resistance R_{eff} between two adjacent nodes by injecting a test current I into any single node on the D-dimensional resistive lattice from the zero-potential boundary at infinity and then extracting another identical test current I from an adjacent node connected to a current sink kept at zero potential. By using Kirchhoff's current law and symmetry, we find that each of the M resistors connected to the original node will receive I/M of the injected current. Similarly, we find that each of the M resistors connected to the adjacent node will receive $-I/M$ of the extracted current in the opposite direction. Therefore, by superposition, the total resulting current flowing in the resistor R connecting the two adjacent nodes will be $2I/M$, which leads to a voltage drop across the resistor R of $V=(2I/M)R$. Thus the effective resistance is

