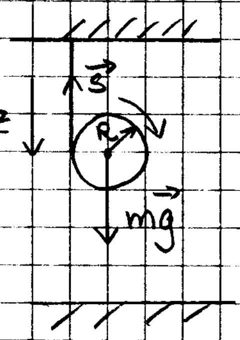
LØSNINGSFORSLAG, SIF 4010
KONTINUASJONSEKSAMEN, AUG. 98

Oppgave 1



a) Betingelsen for ren rullebevegelse

$$\underline{a_{CM} = R\alpha}$$

$$\underline{\underline{v_{CM} = R\omega}}$$

Forflytning i z-retningen av CM:

$$\underline{\underline{\Delta z = R\theta}} \qquad (\theta = 0 \text{ initielt})$$

b) Fra figuren:

$$ma_{CM} = mg - S \qquad (1)$$

Dreiemoment om CM $\Rightarrow$ $S \cdot R = I_{syl} \cdot \alpha = \frac{mR^2}{2} \cdot \frac{a_{CM}}{R}$ (2)

$$\Rightarrow S = \frac{ma_{CM}}{2}$$

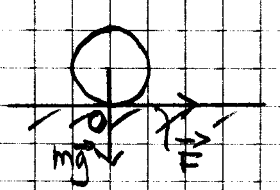
Innsatt i (1): $ma_{CM} = mg - \frac{ma_{CM}}{2}$

$$\Rightarrow \underline{\underline{a_{CM} = \frac{2g}{3}}} \qquad \text{(Vertikalaks.)}$$

Vinkelakselerasjonen: $\underline{\underline{\alpha}} = \frac{a_{CM}}{R} = \underline{\underline{\frac{2g}{3R}}}$

$$\underline{\underline{S}} = \frac{ma_{CM}}{2} = \underline{\underline{\frac{mg}{3}}} \qquad \text{(Snordraget)}$$

c) Friksjonskraft F



I støtet med underlaget er det ingen endring i dreieimpulsen L da $\vec{F}$ og $\vec{G}$ ikke har noen arm ift. O $\Rightarrow$ $\omega_{før} = \omega_{etter}$

Energibevaring ~~i bevegelsen~~ før støtet:

$$mgh = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{syl}\,\omega^2 \qquad (3)$$

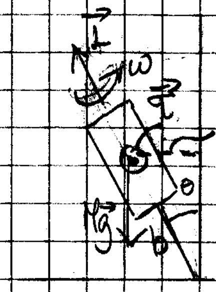
$$v_{CM} = R\omega \qquad (4)$$

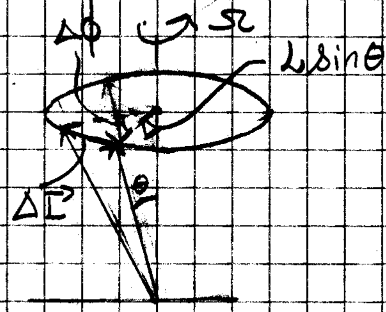
(4) i (3) $\Rightarrow$ $mgh = \frac{1}{2}mR^2\omega^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2 = \frac{3}{4}mR^2\omega^2$

$$\Rightarrow \omega^2 = \frac{4}{3}\,\frac{gh}{R^2}$$

$$\Rightarrow \underline{\omega_{etter} = \omega = \frac{2}{R}\sqrt{\frac{gh}{3}}}$$

d)





Snurrebassens dreieimpuls $L = I_{syl}\,\omega$ da $\Omega \ll \omega$

Dreieimpuls teoremet $\frac{d\vec{L}}{dt} = \vec{\tau}$

$$\Rightarrow \Delta\vec{L} = \vec{\tau}\,\Delta t$$

Fra figuren: $\tau = Mg\,b\sin\theta$

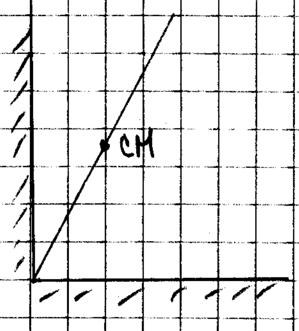
$\vec{\tau} \perp \vec{L} \Rightarrow |\vec{L}| = \text{konst}$, retning endres.

$$\Delta\phi = \Omega\Delta t$$

$$\Delta L = Mg\,b\sin\theta\,\Delta t = L\sin\theta\cdot\Omega\cdot\Delta t$$

$$\Rightarrow \underline{\underline{\Omega}} = \frac{Mgb}{L} = \underline{\underline{\frac{2gb}{R^2\omega}}}$$

Oppgave 2



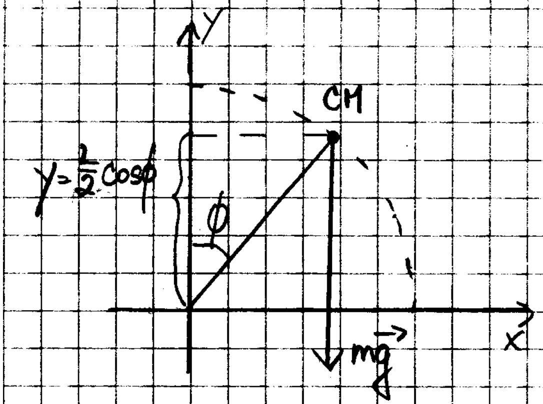
a) Steiners sats:

$$I = I_{CM} + ma^2$$

Her er $a = L/2$ i.f.t. O

$$\Rightarrow \underline{\underline{I_O}} = \frac{mL^2}{12} + \frac{mL^2}{4} = \underline{\underline{\frac{mL^2}{3}}} \quad \text{q.e.d.}$$

b) Om O har vi en ren rotasjonsbevegelse, CM beveger seg langs en sirkel med radius $L/2$.



Energibevarelse:

$$mg\left(\frac{L}{2}-y\right)=\frac{1}{2}I_0\omega^2$$

$$\omega^2=\frac{2mg\cdot 3}{mL^2}\left(\frac{L}{2}-\frac{L}{2}\cos\phi\right)$$

$$\Rightarrow \underline{\underline{\omega=\sqrt{\frac{3g}{L}(1-\cos\phi)}}}\quad(=\dot{\phi})$$

Tyngdekraften er eneste kraft som gir dreiemoment om O:

$$\tau=I_0\alpha=\frac{mL^2}{3}\alpha=mg\frac{L}{2}\cdot\sin\phi$$

$$\Rightarrow \underline{\underline{\alpha=\ddot{\phi}=\frac{3g}{2L}\sin\phi}}$$

(alt. derivasjon)

c) CM følger en sirkel med radius $L/2$

Banefarten $v=\frac{L}{2}\cdot\dot{\phi}$

Baneakselerasjonen $a=\dot{v}=\frac{L}{2}\ddot{\phi}$

I kartesiske koordinater:

$$\underline{\underline{x_{CM}=\frac{L}{2}\sin\phi}}\quad;\quad\underline{\underline{y_{CM}=\frac{L}{2}\cos\phi}}$$

$$\underline{\underline{v_x}}=\dot{x}_{CM}=\frac{L}{2}\cos\phi\cdot\dot{\phi}=\underline{\underline{\frac{L}{2}\cos\phi\sqrt{\frac{3g}{L}(1-\cos\phi)}}}$$

$$\underline{\underline{v_y}}=\dot{y}_{CM}=-\frac{L}{2}\sin\phi\cdot\dot{\phi}=\underline{\underline{-\frac{L}{2}\sin\phi\sqrt{\frac{3g}{L}(1-\cos\phi)}}}$$

$$\underline{\underline{a_x}}=\ddot{x}_{CM}=\frac{L}{2}\left(-\sin\phi\cdot\dot{\phi}\cdot\dot{\phi}+\frac{L}{2}\cos\phi\cdot\ddot{\phi}\right)$$

$$=\frac{L}{2}\sin\phi\left[-\frac{3g}{L}(1-\cos\phi)+\frac{3g}{2L}\cos\phi\right]$$

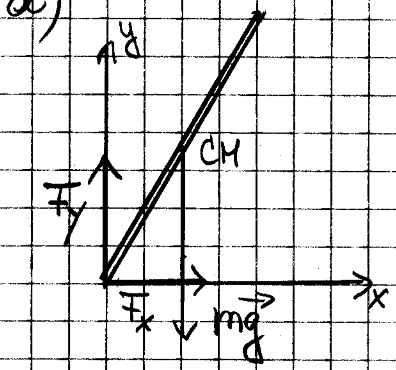
$$=\underline{\underline{\frac{3g}{4}\sin\phi\left[3\cos\phi-2\right]}}$$

$$\underline{\underline{a_y}}=\ddot{y}_{CM}=\frac{L}{2}\left(-\cos\phi\cdot\dot{\phi}\cdot\dot{\phi}-\sin\phi\cdot\ddot{\phi}\right)$$

$$=-\frac{L}{2}\left(\cos\phi\cdot\dot{\phi}^2+\sin\phi\ddot{\phi}\right)$$

$$=\underline{\underline{-\frac{L}{2}\left[\frac{3g}{L}(\cos\phi-\cos^2\phi)+\frac{3g}{2L}\sin^2\phi\right]}}$$

d)



Newtons 2. lov for CM:

$$\Sigma F_x = F_x = ma_x$$

$$\underline{\underline{F_x = \frac{3mg}{4}\sin\phi\left[3\cos\phi - 2\right]}}$$

$$F_y - mg = ma_y$$

$$\underline{F_y = mg - \frac{3mg}{2}\left[\cos\phi - \cos^2\phi + \frac{1}{2}\sin^2\phi\right]}$$

e) $F_x = 0 \Rightarrow \sin\phi\left[3\cos\phi - 2\right] = 0$

For $\phi > 0$ $(\sin\phi > 0)$

får vi $3\cos\phi - 2 = 0$

$$\cos\phi = \frac{2}{3} \Rightarrow \underline{\phi = 48^\circ}$$

$F_y = 0 \Rightarrow 1 - \frac{3}{2}\cos\phi + \frac{3}{2}\cos^2\phi - \frac{3}{4}\sin^2\phi = 0$

$$1 - \frac{3}{2}\cos\phi + \frac{3}{2}\cos^2\phi - \frac{3}{4} + \frac{3}{4}\cos^2\phi = 0$$

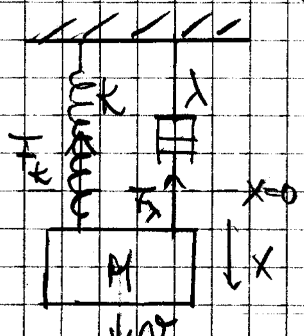
$$9\cos^2\phi - 6\cos\phi + 1 = 0$$

$$\cos\phi = \frac{-(-6) \pm \sqrt{6^2 - 4\cdot 9\cdot 1}}{2\cdot 9} = \frac{1}{3} \Rightarrow \phi = 71^\circ$$

$\Rightarrow$ Staven mister først kontakten med veggen.

Vinkelen er da $\underline{\phi = 48^\circ}$

Oppgave 3



a) Tyngdekraften tar vi ikke hensyn til da likevektsposisjonen svarer til

$F_k^0 = Mg$ (Total F pga. fjær $F_k + F_k^0$)

i. Newtons 2. lov

$$\Rightarrow M\ddot{x} = -F_k - F_\lambda = -kx - \lambda v$$

$$\Rightarrow \underline{\underline{\ddot{x} + \frac{\lambda}{M}\dot{x} + \frac{k}{M}x = 0}}$$

ii. Løsning $x = Ae^{-\gamma t}\cos(\omega t + \alpha)$

$$\underline{\omega_0} = \sqrt{\frac{12{,}5\,kN/m}{5{,}0\,kg}} = \underline{50\,s^{-1}}$$

$$\underline{\underline{\gamma}} = \frac{b}{2M} = \frac{1{,}0}{2\cdot 5{,}0}\,s^{-1} = \underline{\underline{0{,}10\,s^{-1}}}$$

$$\underline{\underline{\omega}} = \sqrt{\omega_0^2 - \gamma^2} \approx \omega_0 = \underline{\underline{50\,s^{-1}}}$$

Startbetingelser:

$$t = 0 \quad x = x_0 \quad ; \quad v = v_0$$

$$t = 0 \Rightarrow \underline{x_0 = A\cos\alpha} \qquad (1)$$

$$\dot{x} = -A\gamma e^{-\gamma t}\cos(\omega t + \alpha) - A\omega e^{-\gamma t}\sin(\omega t + \alpha)$$

$$t = 0 \Rightarrow \dot{x} = \dot{x}_0 = v_0 = A(-\gamma\cos\alpha - \omega\sin\alpha) \qquad (2)$$

$$(2)/(1) \Rightarrow \frac{v_0}{x_0} = -\gamma - \omega\tan\alpha$$

$$\tan\alpha = -\frac{\frac{v_0}{x_0} + \gamma}{\omega} = -\frac{\frac{1}{0{,}020} + 0{,}10}{50}$$

$$\Rightarrow \underline{\alpha} = -45{,}06^\circ \simeq \underline{\underline{-45^\circ}}$$

Fra (1): $\underline{A_0} = \dfrac{x_0}{\cos\alpha} = \dfrac{0{,}020\,m}{0{,}7064} = \underline{\underline{0{,}028\,m}}$

b) i. Bevaring av bevegelsesmengde:

$$mu = (m+M)V \qquad \cdot V = \text{hastighet til fellesle gemet rett etter støtet}$$

$$V = \frac{mu}{M+m} = \frac{0{,}010}{5{,}0+0{,}010}(500\,m/s)$$

$$V = 0{,}998\,m/s \simeq 1{,}0\,m/s$$

Startbetingelsene er nå:

Ved $t = 0$ $\underline{\underline{x = x_0 = 0}}$

$$\underline{\underline{v(t=0) = v_0 = V = 1{,}0\,m/s}}$$

ii. Fra a) $\tan\alpha = \div\infty \Rightarrow \alpha = \div\pi/2$

$$x(t) = Ae^{-\gamma t}\cos(\omega t \div \pi/2) = Ae^{-\gamma t}\sin(\omega t)$$

$t=0$: $v_0 = V = A\omega \Rightarrow A = \frac{V}{\omega} = \frac{mu}{M+m}\cdot\frac{1}{\omega}$

$$\Rightarrow \underline{\underline{x(t) = \frac{m}{m+M}\cdot\frac{u}{\omega}e^{-\gamma t}\sin(\omega t)}}$$

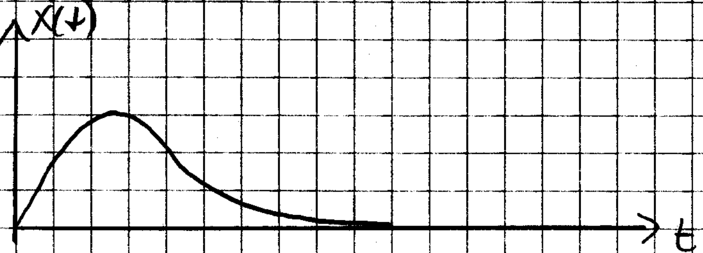
[Velges $u = \div 500$ m/s $\rightarrow x(t) = \div \frac{m}{m+M}\frac{u}{\omega}e^{-\gamma t}\sin(\omega t)$]

iii. Kritisk demping : $\gamma = \omega_0 = \frac{\lambda}{2(M+m)}$

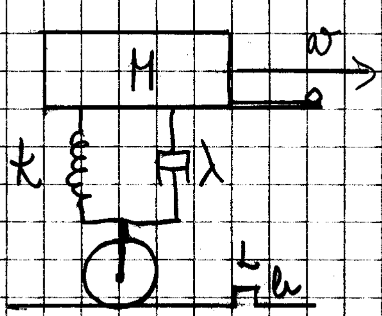
$$\omega_0 = \sqrt{\frac{k}{m+M}} \simeq 50\,\mathrm{s}^{-1}$$

$$\Rightarrow \lambda = 2(M+m)\omega_0 = 2(5{,}0 + 0{,}010)\cdot 50\ \mathrm{kg/s}$$

$$\underline{\underline{\lambda = 500\ \mathrm{Ns/m}}}$$



c)



i. Vertikal hastighet umiddelbart etter at humpen er passert:

$$v_v = a\Delta t = \frac{kh}{M}\cdot\frac{L}{v}$$

Svingebevegelsen er som i b) med starthastighet $v(t=0) = v_v$ og

$x(t=0) = x_0 = 0$

$$\Rightarrow \underline{\underline{x(t) = \frac{khL}{Mv\omega}\cdot e^{-\gamma t}\sin(\omega t)}}$$

ii. Maksimalt utslag er første og største utslag, som finnes ved derivasjon.

$$\dot{x} = -A\gamma e^{-\gamma t}\sin(\omega t) + A\omega e^{-\gamma t}\cos(\omega t)$$

I maks $\dot{x} = 0 \Rightarrow \tan \omega t_0 = \dfrac{\omega}{\gamma}$ (3)

$$\omega_0 = \sqrt{\frac{k}{M}} = \sqrt{\frac{10^5}{600}}\,s^{-1} = 12{,}9\,s^{-1}$$

$$\gamma = \frac{\lambda}{2M} = \frac{8\cdot 10^3}{2\cdot 600}\,s^{-1} = 6{,}67\,s^{-1}$$

$$\omega = \sqrt{\omega_0^2 - \gamma^2} = 11{,}04\,s^{-1}$$

Inn i (3) : $\tan \omega t_0 = 1{,}66$

$$\omega t_0 = 1{,}028 \text{ rad} \quad (58{,}9^\circ)$$

$$t_0 = 0{,}093\,s$$

$$A = \frac{khL}{Mv\cdot\omega} = \frac{1\cdot 10^5 \cdot 0{,}1 \cdot 0{,}1}{600\cdot\frac{50\cdot 10^3}{3600}\cdot 11{,}04}\,m = 0{,}011\,m$$

Maks utslag $\underline{\underline{x(t_0)}} = 0{,}011\,m\; e^{-6{,}67\cdot 0{,}093}\sin(11{,}04\cdot 0{,}093) = \underline{\underline{0{,}0051\,m}}$