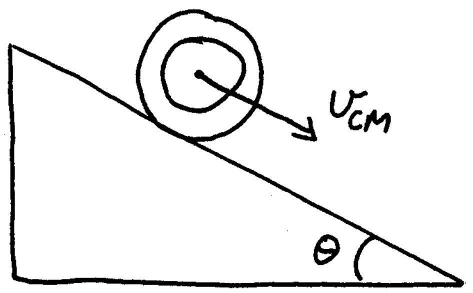
Oppgave 1

Eksamen 14/12 - 98

a) $\underline{\underline{v = R\omega}}$



Total energi: $E = \frac{1}{2}Mv^2 + \frac{1}{2}I\underbrace{\omega^2}_{\left(\frac{v}{R}\right)^2} + Mgy$

$$\Rightarrow \quad E = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2 + Mgy$$

Startbet. $v = v_0$, $y = y_0$

$$E = E_0 = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v_0^2 + Mgy_0$$

$$\Rightarrow \quad v^2 - v_0^2 = \frac{2Mg(y_0 - y)}{M + I/R^2} \quad , \quad M = 5m$$

Treghetsmomentet I:

$$I = \frac{1}{2}(2m)R^2 \cdot 2 + \frac{1}{2}mr^2 = \frac{1}{2}m(4R^2 + r^2)$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + \frac{10mg(y_0 - y)}{5m + 2m + \frac{1}{2}m(r/R)^2}}$$

$$\underline{\underline{v = \sqrt{v_0^2 + \frac{10g(y_0 - y)}{7 + \frac{1}{2}(r/R)^2}}}}$$

Oppg. 1b)

b) Newton's 2. lov (kraftbalanse) gir:

$$Ma = F\cos\beta + Mg\sin\theta - F' \quad ①, \quad M = 5mg$$

hvor F' = friksjonskraft

Videre: $I\alpha = \tau_{tot} = F'R - Fr$ ②

hvor $\alpha > 0$ når jo-jo'en beveger seg nedover skråplanet.

Treghetsmomentet:

$$I = \tfrac{1}{2}m(4R^2 + r^2)$$

① og ② gir:

$$I\alpha = (F\cos\beta + Mg\sin\theta - M\overbrace{R\alpha}^{a})R - Fr$$

$$\Rightarrow (I + MR^2)\alpha = FR\cos\beta + MgR\sin\theta - Fr$$

$$\Rightarrow \quad \alpha = \frac{F(R\cos\beta - r) + 5mg\,R\sin\theta}{\frac{1}{2}m(4R^2 + r^2) + 5mR^2}$$

$$\Rightarrow \quad \alpha = \frac{F(R\cos\beta - r) + 5mg\,R\sin\theta}{7mR^2 + \frac{1}{2}mr^2}$$

c) $R = 2r\ ,\ F = 2mg\ ,\ \theta = 30°$

$$\Rightarrow \quad \alpha = \frac{2mgR\left(\cos\beta - \frac{1}{2}\right) + \frac{5}{2}mgR}{\left(7 + \frac{1}{8}\right)mR^2} = \frac{mgR\left[2\cos\beta + \frac{3}{2}\right]}{mR^2\left(7 + \frac{1}{8}\right)}$$

$$\Rightarrow \quad \underline{\alpha = \frac{56\,g\left[2\cos\beta + \frac{3}{2}\right]}{57R}}$$

$$\alpha = 0 \Rightarrow \cos\beta = -\frac{3}{4} \Rightarrow \underline{\underline{\beta = 138.6°}}$$

$\cos\beta < -\frac{3}{4} \Rightarrow \alpha < 0 \Rightarrow \beta > 138.6°$

$\Rightarrow$ jo-jo'en beveger seg oppover

$\cos\beta > -\frac{3}{4} \Rightarrow \alpha > 0 \Rightarrow \beta < 138.6°$

$\Rightarrow$ jo-jo'en beveger seg nedover

d) Friksjon $F' = \mu N = \mu(5mg\cos\theta - F\sin\beta)$

Også: $\tau = F'R - Fr = I\alpha \Rightarrow F' = \dfrac{I\alpha + Fr}{R}$

$$\Rightarrow \quad \underline{\mu = \frac{(I\alpha + Fr)/R}{5mg\cos\theta - F\sin\beta}}$$

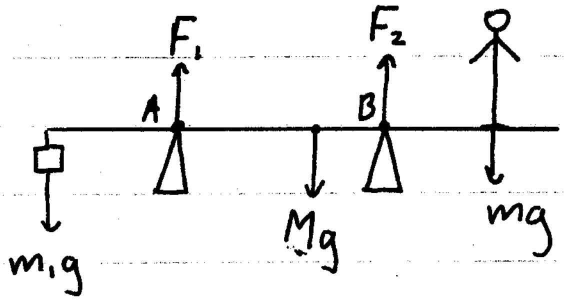
$\alpha = 0,\ r = \frac{1}{2}R,\ F = 2mg\ ,\ \theta = 30° \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\sin\beta = \sin(138.6°) = \sqrt{1 - \cos^2\beta} = \sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4}$

$$\Rightarrow \underline{\underline{\mu}} = \frac{2mg\cdot\frac{1}{2}}{5mg\frac{\sqrt{3}}{2} - 2mg\frac{\sqrt{7}}{4}} = \frac{1}{\frac{5}{2}\sqrt{3} - \frac{\sqrt{7}}{2}} = \underline{\underline{0.33}}$$

Oppgave 2

a) Kraftbalanse:



$$F_1 + F_2 = (m_1 + M + m)g$$

Dreiemoment-bevarelse (om f.eks. B)

$$F_1 \cdot (3,5) + mg \cdot x - m_1 g(1.5+3.5) - Mg \cdot (0.5) = 0$$

b) Bjelken begynner å vippe $\Rightarrow F_1 = 0$

$$\Rightarrow x = \frac{5 \cdot m_1 + 0.5M}{m} = \frac{30 \cdot 5 + 50 \cdot 0.5}{60} m$$

$$\Rightarrow \underline{\underline{x = 2.92 m}}$$

c)

$$I\alpha = \tau = mgx - 5m_1 g - 0.5Mg$$

$$\underline{I} = I_{cm} + MR^2 = M\left[\frac{L^2+h^2}{12} + 0.5^2\right] = \underline{7.0M}$$

$$\Rightarrow \underline{\underline{\alpha}} = \frac{g(mx - 5m_1 - 0.5M)}{7M} = \underline{\underline{1.68x - 4.9}} \; [s^{-2}]$$

$\boxed{\text{Oppgave 3}}$

a) $m\ddot{x} = -kx - \lambda\dot{x} + mg$

$$\Rightarrow \quad m\ddot{x} + \lambda\dot{x} + k\left(x - \frac{mg}{k}\right) = 0$$

Tyngden av massen medfører en forskyvning av likevektsposisjonen.
$\left(x \to x - \frac{mg}{k}\right)$

$$\underline{\underline{m\ddot{x} + \lambda\dot{x} + kx = 0}} \qquad (1)$$

$$x = Ae^{-\gamma t}\cos(\omega t + \alpha)$$

$$\Rightarrow \quad \dot{x} = Ae^{-\gamma t}\left[-\gamma\cos(\omega t + \alpha) - \omega\sin(\omega t + \alpha)\right]$$

$$\Rightarrow \quad \ddot{x} = Ae^{-\gamma t}\left[\gamma^2\cos(\omega t + \alpha) + \gamma\omega\sin(\omega t + \alpha) + \gamma\omega\sin(\omega t + \alpha) - \omega^2\cos(\omega t + \alpha)\right]$$

Innsatt i (1):

Ledd med cos: $m(\gamma^2 - \omega^2) - \lambda\gamma + k = 0$ (2)

Ledd med sin: $m2\gamma\omega - \lambda\omega = 0$ (3)

$$(3) \Rightarrow \quad \underline{\underline{\gamma = \frac{\lambda}{2m}}}$$

$$\text{inn i (2)} \Rightarrow \quad m\frac{\lambda^2}{4m^2} - m\omega^2 - \frac{\lambda^2}{2m} + k = 0$$

$$\Rightarrow \quad \omega^2 = \frac{k}{m} - \frac{\lambda^2}{4m^2}$$

$$\Rightarrow \quad \underline{\underline{\omega = \sqrt{\frac{k}{m} - \left(\frac{\lambda}{2m}\right)^2}}} \quad , \quad \underline{\underline{P = \frac{2\pi}{\omega}}}$$

b) Grensebetingelser

$$x(t=0) = X_0 \Rightarrow A\cos\alpha = X_0$$

$$\dot{x}(t=0) = v_0 \Rightarrow -\gamma A\cos\alpha - \omega A\sin\alpha = v_0$$

$$\cos\alpha = \frac{X_0}{A}$$

$$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \sin\alpha = \sqrt{1-\left(\frac{X_0}{A}\right)^2}$$

$$\Rightarrow -\gamma A\frac{X_0}{A} - \omega A\sqrt{1-\left(\frac{X_0}{A}\right)^2} = v_0$$

$$\Rightarrow \quad -\omega A\sqrt{\quad} = v_0 + \gamma X_0$$

$$\Rightarrow \quad \omega^2 A^2\left(1-\frac{X_0^2}{A^2}\right) = (v_0+\gamma X_0)^2$$

$$\Rightarrow \quad \omega^2 A^2 = \omega^2 X_0^2 + (v_0+\gamma X_0)^2$$

$$\Rightarrow \quad \underline{\underline{A = \sqrt{X_0^2 + \left(\frac{v_0+\gamma X_0}{\omega}\right)^2}}}$$

Videre:

$$\cos\alpha = \frac{X_0}{\sqrt{X_0^2 + \left(\frac{v_0+\gamma X_0}{\omega}\right)^2}}$$

$$\underline{\underline{\alpha = \cos^{-1}\left[\frac{X_0}{\sqrt{X_0^2 + \left(\frac{v_0+\gamma X_0}{\omega}\right)^2}}\right]}}$$

Oppg. 3b forts.)

Fjærkonstanten.

Uten at tyngdekraften virker:

$$m\ddot{x} + \lambda\dot{x} + kx = 0$$

Likevekt for $x = 0$

Med tyngdekraft

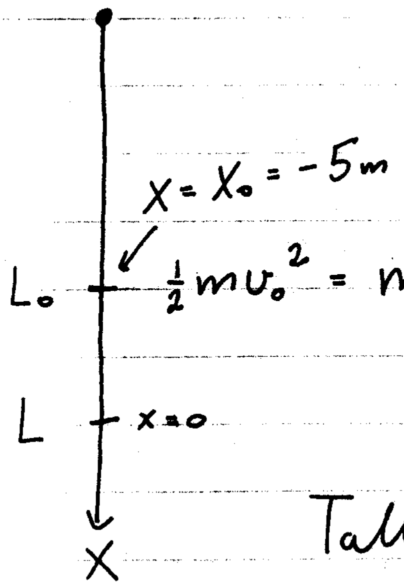
$$m\ddot{x} + \lambda\dot{x} + k\left(x - \frac{mg}{k}\right) = 0$$

Likevekt for $x = \frac{mg}{k}$

$$\Rightarrow \quad \Delta x = \frac{mg}{k} \quad \Rightarrow \quad k = \frac{mg}{\Delta x} = \frac{60 \cdot 9.8}{5}\ \mathrm{N/m}$$

$$\Rightarrow \quad \underline{\underline{k = 117.6\ \mathrm{N/m}}}$$

c)



$$\tfrac{1}{2}mv_0^2 = mgL_0 \quad \Rightarrow \quad v_0 = \sqrt{2gL_0}$$

Tallverdier: $x_0 = -5\,m$

$v_0 = 17.15\ m/s$

$$\gamma = \frac{\lambda}{2m} = \frac{30\ Ns/m}{2 \cdot 60\ kg} = 0.25\ s^{-1}$$

$$\omega = \sqrt{\frac{k}{m} - \gamma^2} = \sqrt{\frac{117.6}{60} - 0.25^2}\ s^{-1} = 1.38\ s^{-1}$$

Dermed fås:

$$\underline{A} = \sqrt{5^2 + \left(\frac{17.15 - 0.25 \cdot 5}{1.38}\right)^2}\ m = \underline{12.56\ m}$$

$$\underline{\alpha} = \cos^{-1}\left(\frac{X_0}{A}\right) = \cos^{-1}\left(\frac{-5}{12.56}\right) = \underline{1.98\ rad.}$$

Laveste punkt finnes ved:

$$\cos(\omega t + \alpha) = 1$$

$$\Rightarrow \omega t + \alpha = 2\pi$$

$$\Rightarrow \underline{t} = \frac{2\pi - \alpha}{\omega} = \frac{6.28 - 1.98}{1.38}\ s = \underline{3.12\ s}$$

Dermed:

$$\underline{X_{lavest}} = 12.56\ e^{-0.25 \cdot 3.12}\ m = \underline{5.76\ m}$$

Avstanden til vannet blir:

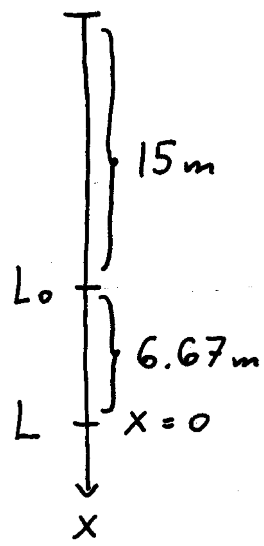
$$\underline{\underline{S}} = [30 - (20 + 5.76 + 1.7)]\ m = \underline{\underline{2.54\ m}}$$

Oppg. 3d)

d) Ny strikkhopper $M = 80\,kg$, $h = 1.8\,m$.

$$M\ddot{x} + \lambda\dot{x} + k\left(x - \frac{Mg}{k}\right) = 0$$

$$x = \frac{Mg}{k} = \frac{80 \cdot 9.8}{117.6}\,m = 6.67\,m \text{ ved likevekt.}$$



$$x_0 = -6.67\,m$$

$$v_0 = 17.15\,m/s \quad \text{som før}$$

$$\underline{\gamma} = \frac{\lambda}{2m} = \frac{30}{2 \cdot 80}\,s^{-1} = \underline{0.188\,s^{-1}}$$

$$\underline{\omega} = \left(\frac{117.6}{80} - 0.188^2\right)^{1/2} s^{-1} = \underline{1.20\,s^{-1}}$$

$$\underline{A} = \sqrt{6.67^2 + \left(\frac{17.15 - 0.188 \cdot 6.67}{1.20}\right)^2}\,m = \underline{14.83\,m}$$

$$\underline{\alpha} = \cos^{-1}\left(\frac{-6.67}{14.83}\right) = \underline{2.04\,rad}$$

Avstand fra vannet til håret på strikkhopperen ved likevekt $S = (30 - 15 - 6.67 - 1.8)\,m$
$\Rightarrow$ $\underline{S = 6.53\,m}$

$x = Ae^{-\gamma t}\cos(\omega t + \alpha)$ har størst verdi for $\cos(\omega t + \alpha) = 1$ $\Rightarrow$ $\omega t + \alpha = 2\pi$

$$\Rightarrow \underline{t} = \frac{2\pi - \alpha}{\omega} = \underline{3.53\,s}$$

$$\underline{X_{maks}} = 14.83 \cdot e^{-0.188 \cdot 3.53}\ m \underline{= 7.64\ m}$$

Siden $X_{maks} > S$ treffer strikkhopperen vannet.

Tiden det tar å treffe vannet finnes tilnærmet: (vet $X = 7.64m \Rightarrow t = 3.53s$)

$$X = Ae^{-\gamma t} \cos(\omega t + \alpha)$$

t	$e^{-\gamma t}$	$\cos(\omega t + \alpha)$	X
3	0.569	0.80	6.75
2.9	0.580	0.72	6.22
2.95	0.574	0.76	6.49 ≈ 6.53

$\Rightarrow t \approx 2.95s$

$$v = \dot{x}(t = 2.95s) = \underbrace{A e^{-\gamma t}}_{8.51}\Big[-\underbrace{\gamma}_{0.188} \underbrace{\cos(\omega t + \alpha)}_{0.76} - \underbrace{\omega}_{1.20} \underbrace{\sin(\omega t + \alpha)}_{-0.65}\Big]$$

$$\Rightarrow \underline{\underline{v = 5.4\ m/s}}$$

Strikkhopperen treffer vannet med hastighet $v \approx 5.4\ m/s$