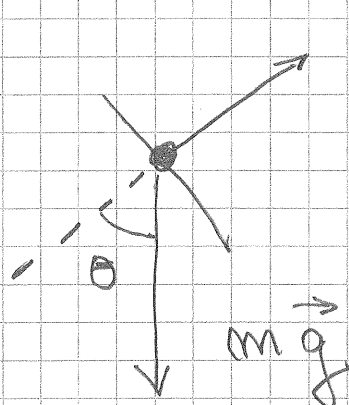
L1

a) Energien bevart i tyngdefeltet ⇒

$$\frac{1}{2}MV^2 = mg\cdot \underbrace{R(1-\cos\theta)}_{\text{"fall-höyde"}} \; ;$$

$$\underline{V(\theta) = \sqrt{2gR(1-\cos\theta)}}$$



Bevegelse $\perp$ flate:

$$mg\cos\theta - N = m\frac{V^2}{R}$$

$$N = mg\cos\theta - m\frac{V^2}{R}$$

$$= mg\cos\theta - 2mg(1-\cos\theta)$$

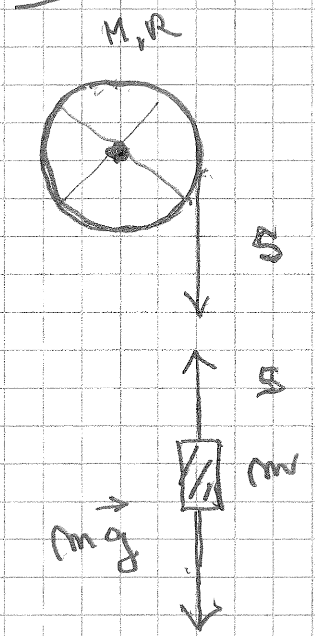
$$= \underline{mg\left[3\cos\theta - 2\right]}$$

b) X letter fra flaten når $N = 0$:

$$\underline{\cos\theta_0 = \frac{2}{3}} \qquad \theta_0 = 48{,}2^\circ$$

Farten er da

$$V(\theta_0) = \sqrt{2gR\left(1-\frac{2}{3}\right)} = \underline{\sqrt{\frac{2}{3}gR}}$$



a) Deler systemet og betrakter lodd og hjul for seg!

Loddets translasjonslov:

$$mg - S = mA \qquad (1)$$

Hjulets rotasjonslov:

$$\underbrace{SR}_{\text{Moment}} = I\alpha \qquad (2)$$

Kinematisk betingelse

$$V = R\omega \Rightarrow A = R\alpha \qquad (3)$$

(2) og (3) $\Rightarrow$ $SR = \underbrace{MR^2}_{I} \cdot \underbrace{\frac{A}{R}}_{\alpha}$): $S = MA$

Innsatt i (1)

$$mg - MA = mA \Rightarrow A = \frac{mg}{m+M}$$

$$S = \frac{mMg}{m+M}$$

Spes: $m \ll M$

$$A \approx \frac{m}{M} g \ll g \qquad S \approx mg$$

Rimelig: En tennisball vil ikke sette ~~et~~ kjerrahjul nevne-verdig mht. bevegelse.

b) Energibevarelse i tyngdefelt:

$$mgH = \underbrace{\tfrac{1}{2} m V^2}_{\text{Transl.}} + \underbrace{\tfrac{1}{2} I \omega^2}_{\text{Rot.}}$$

$$I = MR^2 \qquad \omega = V/R \quad \Rightarrow$$

$$mgH = \tfrac{1}{2} m V^2 + \tfrac{1}{2} M V^2 \quad \Rightarrow$$

$$V = \sqrt{\frac{2mgH}{m+M}}$$

$$K_{lodd} = \tfrac{1}{2} m V^2 = \left(\frac{m}{m+M}\right) mgH$$

$$K_{hjul} = \tfrac{1}{2} M V^2 = \left(\frac{M}{m+M}\right) mgH.$$

Kontroll $m \gg M$.

$$V \approx \sqrt{2gH} \qquad K_{lodd} \simeq mgH \gg K_{hjul}$$

som ventet. "100 kg-lodd med trådsnelle".

Alt: Da A = konst, er

$$V = At \qquad H = \tfrac{1}{2} A t^2 \quad \Rightarrow$$

$$V = \sqrt{2AH} = \sqrt{\frac{2mgH}{m+M}}$$

som ovenfor.

a) Ingen ytre krefter bortsett fra reaksjon fra rot. aksen ∴ Kraftmomentet om aksen lik null ∴ Spinnkomp. langs aksen bevart. Da treghetsmomentet om aksen avtar må vinkelhastigheten øke.

$$\underbrace{I\omega_0 + mRv_0}_{\text{"Før"}} = \underbrace{I\omega + mrv}_{\text{"Etter"}}$$

$$v_0 = R\omega_0 \,, \quad v = R\omega \quad \Rightarrow$$

$$(I + mR^2)\,\omega_0 = (I + mr^2)\,\omega$$

$$\omega = \omega_0 \frac{I + mR^2}{I + mr^2} = \omega_0 \frac{\frac{1}{2}MR^2 + mR^2}{\frac{1}{2}MR^2 + mr^2}$$

$$\omega = \omega_0 \frac{1 + 2\frac{m}{M}}{1 + 2\frac{m}{M}\left(\frac{r}{R}\right)^2}$$

Kontroll: $r \to R \Rightarrow \omega \to \omega_0$.

$m \to 0 \Rightarrow \omega \to \omega_0$

Spes: $r \to 0$ $\quad \omega \to \omega_0\left(1 + \frac{2m}{M}\right)$

b)

$$\Delta K = K(r=0) - K(r=R)$$

$$= \frac{1}{2} I \omega^2(r=0) - \frac{1}{2}\left[I + mR^2\right]\omega_0^2 \Rightarrow$$

$$\Delta K = \frac{1}{4} MR^2 \left(1+\frac{2m}{M}\right)^2 \omega_0^2 - \frac{1}{2}\left[\frac{1}{2}MR^2 + mR^2\right]\omega_0^2 \Rightarrow$$

$$\underline{\Delta K = \frac{1}{2} m \omega_0^2 R^2 \left[1 + \frac{2m}{M}\right]}$$

c)

$$F_s = \frac{m v^2}{r} = m\omega^2 r$$

$$F_s(r) = m\omega_0^2 \left[\frac{1+2\frac{m}{M}}{1+\frac{2m}{M}\frac{r^2}{R^2}}\right]^2 r$$

$$W = \int_{r=R}^{0} (-F_s)\,dr = \int_0^R F_s\,dr$$

$$= m\omega_0^2 \left(1+2\frac{m}{M}\right)^2 \int_0^R \frac{r\,dr}{1+\frac{2m}{M}\frac{r^2}{R^2}}$$

$$Z = \frac{2m}{MR^2} r^2 \Rightarrow r\,dr = \frac{MR^2}{4m}\,dZ$$

$$W = m\omega_0^2 \left(1+\frac{2m}{M}\right)^2 \cdot \frac{MR^3}{4m} \int_0^{2m/M} \frac{dZ}{(1+Z)^2}$$

$$\Big]_Z = \left[-\frac{1}{1+Z}\right]_0^{2m/M} = \frac{2m/M}{1+2m/M} \Rightarrow$$

$$\underline{W = \frac{1}{2} m\omega_0^2 R^2 \left(1+\frac{2m}{M}\right)}$$