

# 1. Introduction

This brochure presents information on the definition and use of the International System of Units, universally known as the SI (from the French *Système international d'unités*), for which the General Conference on Weights and Measures (CGPM) has responsibility. In 1960 the 11<sup>th</sup> CGPM formally defined and established the SI and has subsequently revised it from time to time in response to the requirements of users and advances in science and technology. The most recent and perhaps most significant revision in the SI since its establishment was made in 2018 by the 26<sup>th</sup> CGPM and is documented in this 9<sup>th</sup> edition of the SI brochure. The Metre Convention and its organs, the CGPM, the Comité International des Poids et Mesures (CIPM), the Bureau International des Poids et Mesures (BIPM), and the Consultative Committees are described in Appendix 1.

The SI is a consistent system of units for use in all aspects of life, including international trade, manufacturing, security, health and safety, protection of the environment, and in the basic science that underpins all of these. The system of quantities, underlying the SI, and the equations relating them are based on the present description of nature and are familiar to all scientists, technologists, and engineers.

The definition of the SI units is established in terms of a set of seven defining constants. From the fixed values of these defining constants, expressed in the units of the SI, the complete system of units can be derived. These seven defining constants are the most fundamental feature of the definition of the entire system of units. The specific constants have been identified as the best choice taking into account the previous definition of the SI based on seven base units and the progress in science.

A variety of experimental methods generally described by the Consultative Committees may be used to realize the definitions. Descriptions of realizations are also called “*mises en pratique*”. Realizations may be revised whenever new experiments are developed; for this reason advice on realizing the definitions is not included in this Brochure but is available from the BIPM.

## 1.1 Motivation for the use of defining constants to define the SI

Historically the SI units have been presented in terms of a set of – most recently seven – *base units*. All other units, described as *derived units*, are constructed as products of powers of the base units.

Different types of definitions for the base units have been used: artefacts such as the international prototype (IPK) for the unit kilogram, a specific physical state such as the triple point of water for the unit kelvin, idealized experimental prescriptions as in the case of the ampere and the candela, or constants of nature such as the speed of light for the definition of the unit metre.

To be of any practical use, these units not only have to be defined, but they also have to be realized physically for dissemination. In the case of an artefact, the definition and the realization are equivalent – a path which was pursued by advanced ancient civilizations. Although this is simple and clear, artefacts involve the risk of loss, damage or change. The other types of unit definitions are increasingly abstract or idealized. Here, the realizations are separated conceptually from the definitions, so that the units can, as a matter of principle, be realized independently at any place and at any time. In addition, new superior realizations may be introduced as science and technologies develop, without the need to redefine the unit. These advantages – most obviously seen with the history of the definition

of the metre from artefacts through an atomic reference transition to the fixed numerical value of the speed of light – led to the decision to define all units with the help of defining constants.

The choices of the base units were never unique, but grew historically and became familiar to users of the SI. This description in terms of base and derived units is maintained in the present definition of the SI, but has been reformulated as a consequence of adoption of the defining constants.

## 1.2 Implementation of the SI

The definitions of the SI units as decided by the CGPM represent the highest reference level for measurement traceability to the SI.

The metrology institutes across the world establish the practical realizations of the definitions in order to allow for traceability of measurements to the SI. The Consultative Committees provide the frame for establishing the equivalence of the realizations in order to harmonize the traceability world-wide.

Standardization bodies may specify further details for quantities and units, and rules for their application, where needed by the interested parties. Whenever SI units are concerned, these standards must refer to the definitions by the CGPM. Many of such specifications are listed e. g. in the ISO and IEC 80000 series of international standards.

Individual countries have established rules concerning the use of units by national legislation, either for general use or for specific areas such as commerce, health, public safety, and education. In almost all countries this legislation is based on the SI. The International Organization of Legal Metrology (OIML) is charged with the international harmonization of the technical specifications of this legislation.

## 2. The International System of Units

The value  $Q$  of a quantity is expressed by the product of a number  $\{Q\}$  and a unit  $[Q]$ :

$$Q = \{Q\}[Q] .$$

The unit is simply a particular example of the value of a quantity, defined by convention, which is used as a reference, and the number is the ratio of the value of the quantity to the unit. For a particular quantity different units may be used. For example the value of the speed  $v$  of a particle may be expressed as  $v = 25 \text{ m/s}$  or  $v = 90 \text{ km/h}$ , where metre per second and kilometre per hour are alternative units for the same value of the quantity speed.

Before stating the result of a measurement, it is essential that the quantity being presented is adequately described. This may be simple, such as in the case of the length of a specific steel rod, but can become more complicated when higher accuracy is required and where additional parameters need to be specified, such as the temperature.

When a measurement result of a quantity is reported, the *estimated value* of the measurand (the quantity to be measured), and the *uncertainty* associated with that value are necessary. Both are expressed in the same unit.

## 2.1 Definition of the SI

As for any quantity, the value of a fundamental constant can be expressed as the product of a number and a unit as  $Q = \{Q\} [Q]$ .

The definitions below specify the exact numerical value of each constant when its value is expressed in the corresponding SI unit. By fixing the exact numerical value the unit becomes defined, since the product of the **numerical value**  $\{Q\}$  and the **unit**  $[Q]$  has to equal the **value**  $Q$  of the constant, which is postulated to be invariant.

The seven constants are chosen in such a way that any unit of the SI can be written either through a defining constant itself or through products or ratios of defining constants.

**The International System of Units, the SI, is the system of units in which**

- **the unperturbed ground state hyperfine transition frequency of the caesium 133 atom  $\Delta\nu_{\text{Cs}}$  is 9 192 631 770 Hz,**
- **the speed of light in vacuum  $c$  is 299 792 458 m/s,**
- **the Planck constant  $h$  is  $6.626\,070\,040 \times 10^{-34}$  J s,**
- **the elementary charge  $e$  is  $1.602\,176\,620\,8 \times 10^{-19}$  C,**
- **the Boltzmann constant  $k$  is  $1.380\,648\,52 \times 10^{-23}$  J/K,**
- **the Avogadro constant  $N_{\text{A}}$  is  $6.022\,140\,857 \times 10^{23}$  mol<sup>-1</sup>,**
- **the luminous efficacy  $K_{\text{cd}}$  of monochromatic radiation of frequency  $540 \times 10^{12}$  hertz is 683 lm/W.**

The numerical values of the seven defining constants have no uncertainty.

**Table 1. The seven defining constants of the SI, and the seven corresponding units they define**

Defining constant	Symbol	Numerical value	Unit
hyperfine transition frequency of Cs	$\Delta\nu_{\text{Cs}}$	9 192 631 770	Hz
speed of light in vacuum	$c$	299 792 458	m s <sup>-1</sup>
Planck constant	$h$	$6.626\,070\,040 \times 10^{-34}$	J s
elementary charge	$e$	$1.602\,176\,620\,8 \times 10^{-19}$	C
Boltzmann constant	$k$	$1.380\,648\,52 \times 10^{-23}$	J K <sup>-1</sup>
Avogadro constant	$N_{\text{A}}$	$6.022\,140\,857 \times 10^{23}$	mol <sup>-1</sup>
luminous efficacy	$K_{\text{cd}}$	683	lm W <sup>-1</sup>

Preserving continuity, as far as possible, has always been an essential feature of any changes to the International System of Units. The numerical values of the defining

constants have been chosen to be consistent with the earlier definitions insofar as advances in science and knowledge allow.

### 2.1.1 The nature of the seven defining constants

The nature of the defining constants ranges from fundamental constants of nature to technical constants.

The use of a constant to define a unit disconnects definition from realization. This offers the possibility that completely different or new superior practical realizations can be developed, as technologies evolve without the need to change the definition.

A technical constant like  $K_{\text{cd}}$  refers to a special application. It can be freely chosen in principle, such as to include physiological or other weighting factors by convention. In contrast, a fundamental constant of nature in general will not give this choice but is related to other constants through equations of physics.

The set of the seven defining constants has been chosen such that they provide a most fundamental, stable and universal reference, and simultaneously allow for practical realizations with the smallest uncertainties. The technical conventions and specifications also take historical developments into account.

Both the Planck constant  $h$  and the speed of light in vacuum  $c$  are properly described as fundamental. They determine quantum effects and space-time properties, respectively, and affect all particles and fields equally on all scales and in all environments.

The elementary charge  $e$  corresponds to a coupling strength of the electromagnetic force via the fine-structure constant  $\alpha = e^2/(2c\epsilon_0\hbar)$  where  $\epsilon_0$  is the vacuum electric permittivity or electric constant. Some theories predict a variation of  $\alpha$  over time. The experimental limits of the maximum possible variation in  $\alpha$  are so low, however, that any effect on foreseeable practical measurements can be excluded.

The Boltzmann constant  $k$  corresponds to a conversion factor between the quantities temperature (with unit kelvin) and energy (with unit joule), whereby the numerical value is obtained from historical specifications of the temperature scale. The temperature of a system scales with the thermal energy, but not necessarily with the internal energy of a system. In statistical physics the Boltzmann constant connects the entropy  $S$  with the number  $\Omega$  of quantum-mechanically accessible states,  $S = k \ln \Omega$ .

The caesium frequency  $\Delta\nu_{\text{Cs}}$ , the unperturbed ground-state hyperfine splitting frequency of the caesium-133 atom, has the character of an atomic parameter, which may be affected by the environment, such as by electromagnetic fields. However, the underlying transition is well understood, is stable and is also a good choice as a reference transition under practical considerations. The choice of an atomic parameter like  $\Delta\nu_{\text{Cs}}$  does not disconnect definition and realization like  $h$ ,  $c$ ,  $e$ , or  $k$  does, but specifies the reference.

The Avogadro constant  $N_{\text{A}}$  corresponds to a conversion factor between the quantity amount of substance (with unit mole) and the quantity for counting entities (with unit one, symbol 1). Thus it has the character of a constant of proportionality similar to the Boltzmann constant  $k$ .

The luminous efficacy  $K_{\text{cd}}$  is a technical constant related to a conventional spectral response of the human eye.

## 2.2 Definitions of the SI units

Prior to the 2018 definitions, the SI was defined through seven *base units* from which the *derived units* were constructed as products of powers of the *base units*. Defining the SI by fixing the numerical values of seven defining constants has the effect that this distinction is, in principle, not needed, since all units, *base* as well as *derived units*, may be constructed directly from the defining constants. Nevertheless, the concept of base and derived units is maintained, not only because it is useful and historically well established, but also because it is necessary to maintain consistency with the International System of Quantities (ISQ) defined by ISO/IEC 80 000 series of Standards which specify base and derived quantities to which the SI base and derived units necessarily correspond.

### 2.2.1 Base units

The base units of the SI are listed in Table 2.

**Table 2. SI base units**

Base quantity		Base unit	
Name	Typical symbol	Name	Symbol
time	<i>t</i>	second	s
length	<i>l, x, r, etc.</i>	metre	m
mass	<i>m</i>	kilogram	kg
electric current	<i>I, i</i>	ampere	A
thermodynamic temperature	<i>T</i>	kelvin	K
amount of substance	<i>n</i>	mole	mol
luminous intensity	<i>I<sub>v</sub></i>	candela	cd

The symbols for quantities are generally single letters of the Latin or Greek alphabets, printed in an italic font, and are *recommendations*. The symbols for units are printed in an upright (roman) and are *mandatory*, see chapter 5.

Starting from the definition of the SI in terms of fixed numerical values of the defining constants, definitions of each of the seven base units are deduced by using, as appropriate, one or more of these defining constants to give the following set of definitions:

- **The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency  $\Delta\nu_{\text{Cs}}$ , the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s<sup>-1</sup>**

This definition implies the exact relation  $\Delta\nu_{\text{Cs}} = 9\,192\,631\,770\text{ Hz}$ . Inverting this relation gives an expression for the unit second in terms of the value of the defining constant  $\Delta\nu_{\text{Cs}}$ :

$$1\text{ Hz} = \frac{\Delta\nu_{\text{Cs}}}{9\,192\,631\,770} \quad \text{or} \quad 1\text{ s} = \frac{9\,192\,631\,770}{\Delta\nu_{\text{Cs}}}.$$

The effect of this definition is that the second is equal to the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the unperturbed ground state of the <sup>133</sup>Cs atom.

The reference to an unperturbed atom is intended to make it clear that the definition of the SI second is based on an isolated caesium atom unperturbed by any external field such as ambient black-body radiation.

The second so defined is the unit of proper time in the sense of the general theory of relativity. For the provision of a coordinated time scale the signals of different primary clocks in different locations are combined, which have to be corrected for relativistic caesium frequency shifts (see section 2.2.6).

The CIPM has adopted various secondary representations of the second, based on a selected number of spectral lines of atoms, ions or molecules. The unperturbed frequencies of these lines can be determined with a relative uncertainty not lower than that of the realisation of the second based on the  $^{133}\text{Cs}$  hyperfine splitting, but some can be reproduced with superior stability.

- **The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum  $c$  to be 299 792 458 when expressed in the unit  $\text{m s}^{-1}$ , where the second is defined in terms of the caesium frequency  $\Delta\nu_{\text{Cs}}$ .**

This definition implies the exact relation  $c = 299\,792\,458\,\text{m s}^{-1}$ . Inverting this relation gives an exact expression for the metre in terms of the defining constants  $c$  and  $\Delta\nu_{\text{Cs}}$ :

$$1\,\text{m} = \left( \frac{c}{299\,792\,458} \right) \text{s} = 30.663\,319\dots \frac{c}{\Delta\nu_{\text{Cs}}}.$$

The effect of this definition is that one metre is the length of the path travelled by light in vacuum during a time interval with duration of  $1/299\,792\,458$  of a second.

- **The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant  $h$  to be  $6.626\,070\,040 \times 10^{-34}$  when expressed in the unit  $\text{J s}$ , which is equal to  $\text{kg m}^2 \text{s}^{-1}$ , where the metre and the second are defined in terms of  $c$  and  $\Delta\nu_{\text{Cs}}$ .**

This definition implies the exact relation  $h = 6.626\,070\,040 \times 10^{-34}\,\text{kg m}^2 \text{s}^{-1}$ . Inverting this relation gives an exact expression for the kilogram in terms of the three defining constants  $h$ ,  $\Delta\nu_{\text{Cs}}$  and  $c$ :

$$1\,\text{kg} = \left( \frac{h}{6.626\,070\,040 \times 10^{-34}} \right) \text{m}^2 \text{s} = 1.475\,521\dots \times 10^{40} \frac{h \Delta\nu_{\text{Cs}}}{c^2}.$$

The effect of this definition is to define the unit  $\text{kg m}^2 \text{s}^{-1}$  (the unit of both the physical quantities action and angular momentum). Together with the definitions of the second and the metre this leads to a definition of the unit of mass expressed in terms of the value of the Planck constant  $h$ .

The previous definition of the kilogram fixed the value of the mass of the international prototype of the kilogram,  $m(\mathcal{K})$ , to be equal to one kilogram exactly, and the value of the Planck constant  $h$  had to be determined by experiment. The present definition fixes the value of  $h$  exactly, and the mass of the prototype now has to be determined by experiment.

The number chosen for the numerical value of the Planck constant in this definition is such that at the time of its adoption, the kilogram was equal to the mass of the international prototype,  $m(\mathcal{K}) = 1\,\text{kg}$ , with a relative standard uncertainty of  $2 \times 10^{-8}$ , which was the



standard uncertainty of the combined best estimates of the value of the Planck constant at that time.

Note also that with the present definition primary realizations can be established, in principle, at any point in the mass scale.

- **The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge  $e$  to be  $1.602\,176\,620\,8 \times 10^{-19}$  when expressed in the unit C, which is equal to A s, where the second is defined in terms of  $\Delta\nu_{\text{Cs}}$ .**

This definition implies the exact relation  $e = 1.602\,176\,620\,8 \times 10^{-19}$  A s. Inverting this relation gives an exact expression for the unit ampere in terms of the defining constants  $e$  and  $\Delta\nu_{\text{Cs}}$ :

$$1\text{ A} = \left( \frac{e}{1.602\,176\,620\,8 \times 10^{-19}} \right) \text{s}^{-1} = 6.789\,687\ldots \times 10^8 \Delta\nu_{\text{Cs}} e.$$

The effect of this definition is that one ampere is the electric current corresponding to the flow of  $1/(1.602\,176\,620\,8 \times 10^{-19})$  elementary charges per second.

The previous definition of the ampere based on the force between current carrying conductors had the effect of fixing the value of the vacuum magnetic permeability  $\mu_0$  (also known as magnetic constant) to be exactly  $4\pi \times 10^{-7} \text{ H m}^{-1} = 4\pi \times 10^{-7} \text{ N A}^{-2}$ , where H and N denote the coherent derived units henry and newton, respectively. The new definition of the ampere fixes the value of  $e$  instead of  $\mu_0$ , and as a result  $\mu_0$  must be determined experimentally.

It also follows that since the vacuum electric permittivity  $\epsilon_0$  (also known as electric constant), the characteristic impedance of vacuum  $Z_0$ , and the admittance of vacuum  $Y_0$  are equal to  $1/\mu_0 c^2$ ,  $\mu_0 c$ , and  $1/\mu_0 c$ , respectively, the values of  $\epsilon_0$ ,  $Z_0$ , and  $Y_0$  must now also be determined experimentally, and are affected by the same relative standard uncertainty as  $\mu_0$  since  $c$  is exactly known. The product  $\epsilon_0 \mu_0 = 1/c^2$  and quotient  $Z_0/\mu_0 = c$  remain exact. At the time of adopting the present definition of the ampere,  $\mu_0$  was equal to  $4\pi \times 10^{-7} \text{ H/m}$  with a relative standard uncertainty of less than  $1 \times 10^{-9}$ .

- **The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant  $k$  to be  $1.380\,648\,52 \times 10^{-23}$  when expressed in the unit  $\text{J K}^{-1}$ , which is equal to  $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$ , where the kilogram, metre and second are defined in terms of  $h$ ,  $c$  and  $\Delta\nu_{\text{Cs}}$ .**

This definition implies the exact relation  $k = 1.380\,648\,52 \times 10^{-23} \text{ kg m}^2 \text{s}^{-2} \text{K}^{-1}$ . Inverting this relation gives an exact expression for the kelvin in terms of the defining constants  $k$ ,  $h$  and  $\Delta\nu_{\text{Cs}}$ :

$$1\text{ K} = \left( \frac{1.380\,648\,52}{k} \right) 10^{-23} \text{ kg m}^2 \text{s}^{-2} = 2.266\,665\ldots \frac{\Delta\nu_{\text{Cs}} h}{k}.$$

The effect of this definition is that one kelvin is equal to the change of thermodynamic temperature that results in a change of thermal energy  $kT$  by  $1.380\,648\,52 \times 10^{-23} \text{ J}$ .

The previous definition of the kelvin was based on an exact numerical value assigned to the temperature of the triple point of water  $T_{\text{TPW}}$ , namely 273.16 K (see section 2.5.5). Because the present definition of the kelvin fixes the numerical value of  $k$  instead of  $T_{\text{TPW}}$ , the latter must be determined experimentally. At the time of adopting the present definition  $T_{\text{TPW}}$  was equal to 273.16 K with a relative standard uncertainty of less than  $1 \times 10^{-6}$  based on measurements of  $k$  made prior to the redefinition.

Because of the manner in which temperature scales used to be defined, it remains common practice to express a thermodynamic temperature, symbol  $T$ , in terms of its difference from the reference temperature  $T_0 = 273.15$  K, close to the ice point. This difference is called the Celsius temperature, symbol  $t$ , which is defined by the quantity equation

$$t = T - T_0.$$

The unit of Celsius temperature is the degree Celsius, symbol  $^{\circ}\text{C}$ , which is by definition equal in magnitude to the unit kelvin. A difference or interval of temperature may be expressed in kelvin or in degrees Celsius, the numerical value of the temperature difference being the same in either case. However, the numerical value of a Celsius temperature expressed in degrees Celsius is related to the numerical value of the thermodynamic temperature expressed in kelvin by the relation (see 5.3.1. for an explanation of the notation used here)

$$t/^{\circ}\text{C} = T/\text{K} - 273.15.$$

The kelvin and the degree Celsius are also units of the International Temperature Scale of 1990 (ITS-90) adopted by the CIPM in 1989 in its Recommendation 5 (CI-1989, PV, **57**, 115). Note that the ITS-90 defines two quantities  $T_{90}$  and  $t_{90}$  which are close approximations to the corresponding thermodynamic temperatures  $T$  and  $t$ .

Note also that with the present definition primary realizations of the kelvin can be established, in principle, at any point in the temperature scale.

- **The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles. It is defined by taking the fixed numerical value of the Avogadro constant  $N_A$  to be  $6.022\,140\,857 \times 10^{23}$  when expressed in the unit  $\text{mol}^{-1}$ .**

This definition implies the exact relation  $N_A = 6.022\,140\,857 \times 10^{23} \text{ mol}^{-1}$ . Inverting this relation gives an exact expression for the mole in terms of the defining constant  $N_A$ :

$$1 \text{ mol} = \left( \frac{6.022\,140\,857 \times 10^{23}}{N_A} \right).$$

The effect of this definition is that the mole is the amount of substance of a system that contains  $6.022\,140\,857 \times 10^{23}$  specified elementary entities.

The previous definition of the mole fixed the value of the molar mass of carbon 12,  $M(^{12}\text{C})$ , to be exactly 0.012 kg/mol, whereas now  $M(^{12}\text{C})$  is no longer known exactly and must be determined experimentally. The value chosen for  $N_A$  is such that at the time of adopting the present definition of the mole,  $M(^{12}\text{C})$  was equal to 0.012 kg/mol with a relative standard uncertainty of less than  $1 \times 10^{-9}$ .

The molar mass of any atom or molecule  $X$  may still be obtained from its relative atomic mass from the equation



$$M(X) = A_r(X) [M(^{12}\text{C})/12] = A_r(X) M_u$$

and the molar mass of any atom or molecule  $X$  is also related to the mass of the elementary entity  $m(X)$  by the relation

$$M(X) = N_A m(X) = N_A A_r(X) m_u.$$

In these equations  $M_u$  is the molar mass constant, equal to  $M(^{12}\text{C})/12$ , and  $m_u$  is the unified atomic mass constant, equal to  $m(^{12}\text{C})/12$ . They are related by the Avogadro constant through the relation

$$M_u = N_A m_u.$$

In the name “amount of substance”, the words “of substance” will typically be replaced by words to specify the substance concerned in any particular application, so that one may for example talk of “amount of hydrogen chloride, HCl”, or “amount of benzene, C<sub>6</sub>H<sub>6</sub>”. It is important to always give a precise specification of the entity involved (as emphasized in the definition of the mole); this should preferably be done by giving the molecular chemical formula of the material involved. Although the word “amount” has a more general dictionary definition, the abbreviation of the full name “amount of substance” to “amount” may be used for brevity. This also applies to derived quantities such as “amount-of-substance concentration”, which may simply be called “amount concentration”. In the field of clinical chemistry the name “amount-of-substance concentration” is generally abbreviated to “substance concentration”.

- **The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency  $540 \times 10^{12}$  Hz,  $K_{\text{cd}}$ , to be 683 when expressed in the unit  $\text{lm W}^{-1}$ , which is equal to  $\text{cd sr W}^{-1}$ , or  $\text{cd sr kg}^{-1} \text{ m}^{-2} \text{ s}^3$ , where the kilogram, metre and second are defined in terms of  $h$ ,  $c$  and  $\Delta\nu_{\text{Cs}}$ .**

This definition implies the exact relation  $K_{\text{cd}} = 683 \text{ kg}^{-1} \text{ m}^{-2} \text{ s}^3 \text{ cd sr}$  for monochromatic radiation of frequency  $\nu = 540 \times 10^{12} \text{ Hz}$ . Inverting this relation gives an exact expression for the candela in terms of the defining constants  $K_{\text{cd}}$ ,  $h$  and  $\Delta\nu_{\text{Cs}}$ :

$$1 \text{ cd} = \left( \frac{K_{\text{cd}}}{683} \right) \text{ kg m}^2 \text{ s}^{-3} \text{ sr}^{-1} = 2.614\,830\dots \times 10^{10} (\Delta\nu_{\text{Cs}})^2 h K_{\text{cd}}.$$

The effect of this definition is that one candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency  $540 \times 10^{12} \text{ Hz}$  and has a radiant intensity in that direction of  $(1/683) \text{ W/sr}$ . The definition of the steradian is given under Table 4, below.

### 2.2.2 Practical realization of SI units

The highest-level experimental methods used for the realization of units using the equations of physics are known as primary methods. The essential characteristic of a primary method is that it allows a quantity to be measured in a particular unit by using only measurements of quantities that do not involve that unit. In the present formulation of the SI the basis of the definitions is different from that used previously, so new methods may be used for the practical realization of SI units.

Instead of each definition specifying a particular condition or physical state, which sets a fundamental limit to the accuracy of realization, it is now open to any user to choose any convenient equation of physics that links the defining constants to the quantity intended to be measured. This is a much more general way of defining the basic units of measurement. It is not limited by today's science or technology but future developments may lead to different ways of realizing units to higher accuracy. Defined in this way, there is, in principle, no limit to the accuracy with which a unit might be realized. The exception remains the definition of the second, in which the original microwave transition of caesium must remain, for the time being, the basis of the definition. For a more complete explanation of the realization of SI units see Appendix I Part 1.

### 2.2.3 Dimensions of quantities

Physical quantities can be organised in a system of dimensions, where the system used is decided by convention. Each of the seven base quantities used in the SI is regarded as having its own dimension. The symbols used for the base quantities, and the symbols used to denote their dimension, are as follows.

**Table 3. Base quantities and dimensions used in the SI**

Base quantity	Typical symbol for quantity	Symbol for dimension
time	$t$	T
length	$l, x, r$ , etc.	L
mass	$m$	M
electric current	$I, i$	I
thermodynamic temperature	$T$	Θ
amount of substance	$n$	N
luminous intensity	$I_v$	J

All other quantities, with the exception of counts, are derived quantities, which may be written in terms of base quantities according to the equations of physics. The dimensions of the derived quantities are written as products of powers of the dimensions of the base quantities using the equations that relate the derived quantities to the base quantities. In general the dimension of any quantity  $Q$  is written in the form of a dimensional product,

$$\dim Q = T^{\alpha} L^{\beta} M^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}$$

where the exponents  $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$ , and  $\eta$ , which are generally small integers which can be positive, negative, or zero, are called the dimensional exponents.

There are quantities  $Q$  for which the defining equation is such that all of the dimensional exponents in the equation for the dimension of  $Q$  are zero. This is true in particular for any quantity that is defined as the ratio of two quantities of the same kind. For example, the refractive index is the ratio of two speeds, and the relative permittivity is the ratio of the permittivity of a dielectric medium to that of free space. Such quantities are simply numbers. The associated unit is the unit one, symbol 1, although this is rarely explicitly written (see 5.3.7).

There are also quantities that cannot be described in terms of the seven base quantities of the SI at all, but have the nature of a count. Examples are a number of molecules, a number of cellular or biomolecular entities (e.g. copies of a particular nucleic acid sequence), or degeneracy in quantum mechanics. Counting quantities are also quantities with the associated unit one.

The unit one is the neutral element of any system of units – necessarily and present automatically. There is no requirement to introduce it formally by decision. Therefore, a formal traceability to the SI can be established through appropriate, validated measurement procedures.

Plane and solid angles, when expressed in radians and steradians respectively, are in effect also treated within the SI as quantities with the unit one (see also section 3.5.8). The symbols rad and sr are explicitly written where appropriate, in order to emphasise that, for radians or steradians, the quantity being considered is or involves the plane angle or solid angle, respectively, or, for steradians, to emphasise e. g. the distinction between fluxes and intensities in radiometry and photometry. However, it is long-established practice in mathematics and across all areas of science to make use of  $\text{rad} = 1$  and  $\text{sr} = 1$ . For historical reasons the radian and steradian are treated as derived units, as described below.

It is especially important to have a clear description of any quantity with unit one (see also section 5.3.7) that is expressed as a ratio of quantities of the same kind (length ratios, amount fractions, etc.) or as a count (number of photons, decays, etc.).

#### 2.2.4 Derived units

Derived units are defined as products of powers of the base units. When the numerical factor of this product is one, the derived units are called *coherent derived units*. The base and coherent derived units of the SI form a coherent set, designated the *set of coherent SI units*. The word “coherent” here means that equations between the numerical values of quantities take exactly the same form as the equations between the quantities themselves.

Some of the coherent derived units in the SI are given special names. Table 4 lists 22 SI units with special names. Together with the seven base units (Table 2) they form the core of the set of SI units. All other SI units are combinations of some of these 29 units.

It is important to note that any of the seven base units and 22 SI units with special names can be constructed directly from the seven defining constants. In fact, the units of the seven defining constants include both base and derived units.

The CGPM has adopted a series of prefixes for use in forming the decimal multiples and submultiples of the coherent SI units (see chapter 3). They are convenient for expressing the values of quantities that are much larger than or much smaller than the coherent unit. However when prefixes are used with SI units, the resulting units are no longer coherent, because the prefix introduces a numerical factor other than one. Prefixes may be used with any of the 29 SI units with special names with the exception of the base unit kilogram, which is further explained in chapter 3.

**Table 4. The 22 SI units with special names and symbols**

Derived quantity	Special name of unit	Unit expressed in terms of base units <sup>(a)</sup>	Unit expressed in terms of other SI units
------------------	-------------------------	---	--

plane angle	radian <sup>(b)</sup>	rad = m/m	
solid angle	steradian <sup>(c)</sup>	sr = m <sup>2</sup> /m <sup>2</sup>	
frequency	hertz <sup>(d)</sup>	Hz = s <sup>-1</sup>	
force	newton	N = kg m s <sup>-2</sup>	
pressure, stress	pascal	Pa = kg m <sup>-1</sup> s <sup>-2</sup>	
energy, work, amount of heat	joule	J = kg m <sup>2</sup> s <sup>-2</sup>	N m
power, radiant flux	watt	W = kg m <sup>2</sup> s <sup>-3</sup>	J/s
electric charge	coulomb	C = A s	
electric potential difference <sup>(e)</sup>	volt	V = kg m <sup>2</sup> s <sup>-3</sup> A <sup>-1</sup>	W/A
capacitance	farad	F = kg <sup>-1</sup> m <sup>-2</sup> s <sup>4</sup> A <sup>2</sup>	C/V
electric resistance	ohm	Ω = kg m <sup>2</sup> s <sup>-3</sup> A <sup>-2</sup>	V/A
electric conductance	siemens	S = kg <sup>-1</sup> m <sup>-2</sup> s <sup>3</sup> A <sup>2</sup>	A/V
magnetic flux	weber	Wb = kg m <sup>2</sup> s <sup>-2</sup> A <sup>-1</sup>	V s
magnetic flux density	tesla	T = kg s <sup>-2</sup> A <sup>-1</sup>	Wb/m <sup>2</sup>
inductance	henry	H = kg m <sup>2</sup> s <sup>-2</sup> A <sup>-2</sup>	Wb/A
Celsius temperature	degree Celsius <sup>(f)</sup>	°C = K	
luminous flux	lumen	lm = cd sr	cd sr
illuminance	lux	lx = cd sr m <sup>-2</sup>	lm/m <sup>2</sup>
activity referred to a radionuclide <sup>(d, g)</sup>	becquerel	Bq = s <sup>-1</sup>	
absorbed dose, kerma	gray	Gy = m <sup>2</sup> s <sup>-2</sup>	J/kg
dose equivalent	sievert <sup>(h)</sup>	Sv = m <sup>2</sup> s <sup>-2</sup>	J/kg
catalytic activity	katal	kat = mol s <sup>-1</sup>	

- (a) The order of symbols for base units in this Table is different from that in the 8th edition following a decision by the CCU at its 21st meeting in 2013 to return to the original order in Resolution 12 of the 11th CGPM in 1960 in which newton was written kg m s<sup>-2</sup>, the joule as kg m<sup>2</sup> s<sup>-2</sup> and J s as kg m<sup>2</sup> s<sup>-1</sup>. The intention was to reflect the underlying physics of the corresponding quantity equations although for some more complex derived units this may not be possible.
- (b) The radian is the coherent unit for plane angle. One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius. It is also the unit for phase angle. For periodic phenomena, the phase angle increases by  $2\pi$  rad in one period. The radian was formerly an SI supplementary unit, but this category was abolished in 1995.
- (c) The steradian is the coherent unit for solid angle. One steradian is the solid angle subtended at the center of a sphere by an area of the surface that is equal to the squared radius. Like the radian, the steradian was formerly an SI supplementary unit.
- (d) The hertz shall only be used for periodic phenomena, and the becquerel shall only be used for stochastic processes in activity referred to a radionuclide.
- (e) Electric potential difference is also called “voltage” in many countries, as well as “electric tension” or simply “tension” in some countries.
- (f) The degree Celsius is used to express Celsius temperatures. The numerical value of a temperature difference or temperature interval is the same when expressed in either degrees Celsius or in kelvin.
- (g) Activity referred to a radionuclide is sometimes incorrectly called radioactivity.
- (h) See CIPM Recommendation 2 on the use of the sievert (PV, 2002, **70**, 205).

The seven base units and 22 units with special names and symbols may be used in combination to express the units of other derived quantities. Since the number of quantities is without limit, it is not possible to provide a complete list of derived quantities and derived units. Table 5 lists some examples of derived quantities and the corresponding coherent derived units expressed in terms of base units. In addition, Table 6 lists examples of coherent derived units whose names and symbols also include derived units. The complete set of SI units includes both the coherent set and the multiples and submultiples formed by using the SI prefixes.

**Table 5. Examples of coherent derived units in the SI expressed in terms of base units**

Derived quantity	Typical symbol of quantity	Derived unit expressed in terms of base units
area	$A$	$\text{m}^2$
volume	$V$	$\text{m}^3$
speed, velocity	$v$	$\text{m/s}$
acceleration	$a$	$\text{m/s}^2$
wavenumber	$\sigma, \nu$	$\text{m}^{-1}$
density, mass density	$\rho$	$\text{kg/m}^3$
surface density	$\rho_A$	$\text{kg/m}^2$
specific volume	$v$	$\text{m}^3/\text{kg}$
current density	$j$	$\text{A/m}^2$
magnetic field strength	$H$	$\text{A/m}$
amount of substance concentration	$c$	$\text{mol/m}^3$
mass concentration	$\rho, \gamma$	$\text{kg/m}^3$
luminance	$L_v$	$\text{cd/m}^2$

**Table 6. Examples of SI coherent derived units whose names and symbols include SI coherent derived units with special names and symbols**

Derived quantity	Name of coherent derived unit	Symbol	Derived unit expressed in terms of base units
dynamic viscosity	pascal second	$\text{Pa s}$	$\text{kg m}^{-1} \text{s}^{-1}$
moment of force	newton metre	$\text{N m}$	$\text{kg m}^2 \text{s}^{-2}$
surface tension	newton per metre	$\text{N/m}$	$\text{kg s}^{-2}$
angular velocity, angular frequency	radian per second	$\text{rad/s}$	$\text{s}^{-1}$
angular acceleration	radian per second squared	$\text{rad/s}^2$	$\text{s}^{-2}$
heat flux density, irradiance	watt per square metre	$\text{W/m}^2$	$\text{kg s}^{-3}$
heat capacity, entropy	joule per kelvin	$\text{J/K}$	$\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$
specific heat capacity,	joule per kilogram kelvin	$\text{J/(kg K)}$	$\text{m}^2 \text{s}^{-2} \text{K}^{-1}$

specific entropy			
specific energy	joule per kilogram	J/kg	$\text{m}^2 \text{s}^{-2}$
thermal conductivity	watt per metre kelvin	W/(m K)	$\text{kg m s}^{-3} \text{K}^{-1}$
energy density	joule per cubic metre	J/m <sup>3</sup>	$\text{kg m}^{-1} \text{s}^{-2}$
electric field strength	volt per metre	V/m	$\text{kg m s}^{-3} \text{A}^{-1}$
electric charge density	coulomb per cubic metre	C/m <sup>3</sup>	$\text{A s m}^{-3}$
surface charge density	coulomb per square metre	C/m <sup>2</sup>	$\text{A s m}^{-2}$
electric flux density, electric displacement	coulomb per square metre	C/m <sup>2</sup>	$\text{A s m}^{-2}$
permittivity	farad per metre	F/m	$\text{kg}^{-1} \text{m}^{-3} \text{s}^4 \text{A}^2$
permeability	henry per metre	H/m	$\text{kg m s}^{-2} \text{A}^{-2}$
molar energy	joule per mole	J/mol	$\text{kg m}^2 \text{s}^{-2} \text{mol}^{-1}$
molar entropy, molar heat capacity	joule per mole kelvin	J/(mol K)	$\text{kg m}^2 \text{s}^{-2} \text{mol}^{-1} \text{K}^{-1}$
exposure (x- and $\gamma$ -rays)	coulomb per kilogram	C/kg	$\text{A s kg}^{-1}$
absorbed dose rate	gray per second	Gy/s	$\text{m}^2 \text{s}^{-3}$
radiant intensity	watt per steradian	W/sr	$\text{kg m}^2 \text{s}^{-3}$
radiance	watt per square metre steradian	W/(sr m <sup>2</sup> )	$\text{kg s}^{-3}$
catalytic activity concentration	katal per cubic metre	kat/m <sup>3</sup>	$\text{mol s}^{-1} \text{m}^{-3}$

It is important to emphasize that each physical quantity has only one coherent SI unit, even though this unit can be expressed in different forms by using some of the special names and symbols.

The converse, however, is not true, because in general several different quantities may share the same SI unit. For example, for the quantity heat capacity as well as for the quantity entropy the SI unit is joule per kelvin. Similarly for the base quantity electric current as well as the derived quantity magnetomotive force the SI unit is the ampere. It is therefore important not to use the unit alone to specify the quantity. This applies not only to technical texts, but also, for example, to measuring instruments (i.e. the instrument read-out needs to indicate both the unit and the quantity measured).

In practice, with certain quantities, preference is given to the use of certain special unit names, to facilitate the distinction between different quantities having the same dimension. When using this freedom one may recall the process by which this quantity is defined. For example the quantity torque is the cross product of distance and force, suggesting the unit newton metre, even though it has the same dimension as energy, and could be expressed using the unit joule.

The SI unit of frequency is hertz, the SI unit of angular velocity and angular frequency is radian per second, and the SI unit of activity is becquerel, implying counts per second. Although it is formally correct to write all three of these units as the reciprocal second, the use of the different names emphasizes the different nature of the quantities concerned. It is especially important to carefully distinguish frequencies from angular frequencies, because by definition their numerical values differ by a factor<sup>1</sup> of  $2\pi$ . Ignoring this fact may cause an error of  $2\pi$ . Note that in some countries frequency values are conventionally expressed using "cycle/s" or "cps" instead of the SI unit Hz, although "cycle" and "cps" are not units

<sup>1</sup> see ISO 80000-3 for details



in the SI. Note also that it is common, although not recommended, to use the term frequency for quantities expressed in rad/s. Because of this, it is recommended that quantities called 'frequency', 'angular frequency', and 'angular velocity' always be given explicit units of Hz or rad/s and not  $s^{-1}$ .

In the field of ionizing radiation, the SI unit becquerel rather than the reciprocal second is used. The SI units gray and the sievert are used for absorbed dose and dose equivalent, respectively, rather than joule per kilogram. The special names becquerel, gray and sievert were specifically introduced because of the dangers to human health that might arise from mistakes involving the units reciprocal second and joule per kilogram, in case the latter units were incorrectly taken to identify the different quantities involved.

Special care must be taken when expressing temperatures or temperature differences, respectively. A temperature difference of 1 K equals that of 1 °C, but for an absolute temperature the difference of 273.15 K must be taken into account. The unit degree Celsius is only coherent when expressing temperature differences.

### **2.2.5 Units for quantities that describe biological and physiological effects**

Four of the SI units listed in tables 2 and 4 include physiological weighing factors: candela, lumen, lux, and sievert.

Lumen and lux are derived from the base unit candela. Like the candela they carry information about human vision. The candela was established as a base unit in 1964, acknowledging the importance of light in daily life.

Ionizing radiation deposits energy in irradiated matter. The ratio of deposited energy to mass is termed absorbed dose  $D$ . As decided by the CIPM in 2002, the quantity dose equivalent  $H = Q D$  is the product of the absorbed dose  $D$  and a numerical quality factor  $Q$  that takes into account the biological effectiveness of the radiation and is dependent on the energy and the type of radiation.

There are units for quantities that describe biological effects and involve weighting factors, which are not SI units. Two examples are given here:

Sound causes pressure fluctuations in the air, superimposed on the normal atmospheric pressure, that are sensed by the human ear. The sensitivity of the ear depends on the frequency of the sound, but it is not a simple function of either the pressure changes or the frequency. Therefore frequency-weighted quantities are used in acoustics to approximate the way in which sound is perceived. They are used, for example, for measurements concerning protection against hearing damage. The effect of ultrasonic acoustic waves poses similar concerns in medical diagnosis and therapy.

There is a class of units for quantifying the biological activity of certain substances used in medical diagnosis and therapy that cannot yet be defined in terms of the units of the SI. This lack of definition is because the mechanism of the specific biological effect of these substances is not yet sufficiently well understood for it to be quantifiable in terms of physico-chemical parameters. In view of their importance for human health and safety, the World Health Organization (WHO) has taken responsibility for defining WHO International Units (IU) for the biological activity of such substances.

### **2.2.6 SI units in the framework of the general theory of relativity**

The practical realization of a unit and the process of comparison require a set of equations within a framework of a theoretical description. In some cases, these equations include relativistic effects.

For frequency standards it is possible to establish comparisons at a distance by means of electromagnetic signals. To interpret the results the general theory of relativity is required, since it predicts, among other things, a relative frequency shift between standards of about 1 part in  $10^{16}$  per metre of altitude difference at the surface of the earth. Effects of this magnitude must be corrected for when comparing the best frequency standards.

When practical realizations are compared locally, i.e. in a small space-time domain, effects due to the space-time curvature described by the general theory of relativity can be neglected. If realizations share the same space-time coordinates (e.g. the same motion and acceleration or gravitational field, respectively), relativistic effects may be neglected entirely.

### 3 Decimal multiples and sub-multiples of SI units

Decimal multiples and submultiples ranging from  $10^{24}$  to  $10^{-24}$  are provided for use with the SI units. The names and symbols of these multiple and sub-multiple prefixes are presented in Table 7 below.

Prefix symbols are printed in upright typeface, as are unit symbols, regardless of the typeface used in the surrounding text, and are attached to unit symbols without a space between the prefix symbol and the unit symbol. With the exception of da (deca), h (hecto), and k (kilo), all multiple prefix symbols are upper-case letters, and all submultiple prefix symbols are lowercase letters. All prefix names are printed in lowercase letters, except at the beginning of a sentence.

**Table 7. SI prefixes**

Factor	Name	Symbol	Factor	Name	Symbol
$10^1$	deca	da	$10^{-1}$	deci	d
$10^2$	hecto	h	$10^{-2}$	centi	c
$10^3$	kilo	k	$10^{-3}$	milli	m
$10^6$	mega	M	$10^{-6}$	micro	$\mu$
$10^9$	giga	G	$10^{-9}$	nano	n
$10^{12}$	tera	T	$10^{-12}$	pico	p
$10^{15}$	peta	P	$10^{-15}$	femto	f
$10^{18}$	exa	E	$10^{-18}$	atto	a
$10^{21}$	zetta	Z	$10^{-21}$	zepto	z
$10^{24}$	yotta	Y	$10^{-24}$	yocto	y

The SI prefixes refer strictly to powers of 10. They should not be used to indicate powers of 2 (for example, one kilobit represents 1000 bits and not 1024 bits).

The names and symbols for prefixes to be used with powers of 2 are recommended as follows:

kibi	Ki	$2^{10}$
mebi	Mi	$2^{20}$
gibi	Gi	$2^{30}$
tebi	Ti	$2^{40}$
pebi	Pi	$2^{50}$
exbi	Ei	$2^{60}$
zebi	Zi	$2^{70}$
yobi	Yi	$2^{80}$

The grouping formed by a prefix symbol attached to a unit symbol constitutes a new inseparable unit symbol (forming a multiple or submultiple of the unit concerned) that can

be raised to a positive or negative power and that can be combined with other unit symbols to form compound unit symbols.

*Examples:* pm (picometre), mmol (millimole), GΩ (gigaohm), THz (terahertz)

$$2.3 \text{ cm}^3 = 2.3 (\text{cm})^3 = 2.3 (10^{-2} \text{ m})^3 = 2.3 \times 10^{-6} \text{ m}^3$$

$$1 \text{ cm}^{-1} = 1 (\text{cm})^{-1} = 1 (10^{-2} \text{ m})^{-1} = 10^2 \text{ m}^{-1} = 100 \text{ m}^{-1}.$$

Similarly prefix names are also inseparable from the unit names to which they are attached. Thus, for example, millimetre, micropascal, and meganewton are single words.

Compound prefix symbols, i.e. prefix symbols formed by the juxtaposition of two or more prefix symbols, are not permitted. This rule also applies to two or more compound prefix names.

Prefix symbols can neither stand alone nor be attached to the number 1, the symbol for the unit one. Similarly prefix names cannot be attached to the name of the unit one, that is, to the word “one”.

The kilogram is the only coherent SI unit, whose name and symbol, for historical reasons, include a prefix. Names and symbols for decimal multiples and sub-multiples of the unit of mass are formed by attaching prefix names and symbols to the unit name “gram” and the unit symbol “g” respectively. For example  $10^{-6} \text{ kg}$  is written as milligram, mg, not as microkilogram,  $\mu\text{kg}$ .

## 4 Non-SI units that are accepted for use with the SI

The SI provides the internationally agreed reference in terms of which all other units are defined. The coherent SI units have the important advantage that unit conversions are not required when inserting particular values for quantities into quantity equations.

Nonetheless it is recognized that some non-SI units are widely used and are expected to continue to be used for many years. Therefore, the CIPM has accepted some non-SI units for use with the SI; these are listed in Table 8. If these units are used it should be understood that some advantages of the SI are lost. The SI prefixes can be used with several of these units, but for example not with the non-SI units of time.

**Table 8. Non-SI units accepted for use with the SI Units**

Quantity	Name of unit	Symbol for unit	Value in SI units
time	minute	min	1 min = 60 s
	hour	h	1 h = 60 min = 3600 s
	day	d	1 d = 24 h = 86 400 s
length	astronomical unit <sup>(a)</sup>	au	1 au = 149 597 870 700 m
plane and			
phase angle	degree	°	1° = (π/180) rad
	minute	'	1' = (1/60)° = (π/10 800) rad

	second <sup>(b)</sup>	"	1" = (1/60)' = ( $\pi$ /648 000) rad
area	hectare <sup>(c)</sup>	ha	1 ha = 1 hm <sup>2</sup> = 10 <sup>4</sup> m <sup>2</sup>
volume	litre <sup>(d)</sup>	l, L	1 l = 1 L = 1 dm <sup>3</sup> = 10 <sup>3</sup> cm <sup>3</sup> = 10 <sup>-3</sup> m <sup>3</sup>
mass	tonne <sup>(e)</sup>	t	1 t = 10 <sup>3</sup> kg
	dalton <sup>(f)</sup>	Da	1 Da = 1.660 538 86 (28) $\times$ 10 <sup>-27</sup> kg
energy	electronvolt <sup>(g)</sup>	eV	1 eV = 1.602 176 620 8 $\times$ 10 <sup>-19</sup> J
logarithmic ratio quantities	neper <sup>(h)</sup>	Np	see text
	bel <sup>(h)</sup>	B	
	decibel <sup>(h)</sup>	dB	

- (a) As decided at the XXVIII General Assembly of the International Astronomical Union (Resolution B2, 2012).
- (b) For some applications such as in astronomy, small angles are measured in arcseconds (i.e. seconds of plane angle), denoted as or ", milliarcseconds, microarcseconds, and picoarcseconds, denoted mas,  $\mu$ as, and pas, respectively, where arcsecond is an alternative name for second of plane angle.
- (c) The unit hectare, and its symbol ha, were adopted by the CIPM in 1879 (PV, 1879, 41). The hectare is used to express land area.
- (d) The litre, and the symbol lower-case l, were adopted by the CIPM in 1879 (PV, 1879, 41). The alternative symbol, capital L, was adopted by the 16th CGPM (1979, Resolution 6; CR, 101 and *Metrologia*, 1980, **16**, 56-57) in order to avoid the risk of confusion between the letter l (el) and the numeral 1 (one).
- (e) The tonne, and its symbol t, were adopted by the CIPM in 1879 (PV, 1879, 41). This unit is sometimes referred to as "metric ton" in some English-speaking countries.
- (f) The dalton (Da) and the unified atomic mass unit (u) are alternative names (and symbols) for the same unit, equal to 1/12 of the mass of a free carbon 12 atom, at rest and in its ground state.
- (g) The electronvolt is the kinetic energy acquired by an electron in passing through a potential difference of one volt in vacuum. The electronvolt is often combined with the SI prefixes.
- (h) In using these units it is important that the nature of the quantity be specified, and that any reference value used be specified.

Table 8 also includes the units of logarithmic ratio quantities, the neper, bel, and decibel. They are used to convey information on the nature of the logarithmic ratio quantity concerned. The neper, Np, is used to express the values of quantities whose numerical values are based on the use of the neperian (or natural) logarithm,  $\ln = \log_e$ . The bel and the decibel, B and dB, where 1 dB = (1/10) B, are used to express the values of logarithmic ratio quantities whose numerical values are based on the decadic logarithm,  $\lg = \log_{10}$ . The statement  $L_X = m$  dB = ( $m/10$ ) B (where  $m$  is a number) is interpreted to mean that  $m = 10 \lg(X/X_0)$ . The units neper, bel, and decibel have been accepted by the CIPM for use with the International System, but are not SI units.

There are many more non-SI units, which are either of historical interest, or are still used in specific fields (for example, the barrel of oil) or in particular countries (the inch, foot, and yard). The CIPM can see no case for continuing to use these units in modern scientific and technical work. However, it is clearly a matter of importance to be able to recall the relation of these units to the corresponding SI units, and this will continue to be true for many years. The CIPM has therefore decided to compile a list of the conversion factors to the SI for such units and to make this available on the BIPM website.

## 5 Writing unit symbols and names, and expressing the values of quantities

General principles for the writing of unit symbols and numbers were first given by the 9th CGPM (1948, Resolution 7). These were subsequently elaborated by ISO, IEC, and other international bodies. As a consequence, there now exists a general consensus on how unit symbols and names, including prefix symbols and names as well as quantity symbols should be written and used, and how the values of quantities should be expressed. Compliance with these rules and style conventions, the most important of which are presented in this chapter, supports the readability of scientific and technical papers.

### 5.1 Unit symbols

Unit symbols are printed in upright type regardless of the type used in the surrounding text. They are printed in lower-case letters unless they are derived from a proper name, in which case the first letter is a capital letter.

An exception, adopted by the 16th CGPM (1979, Resolution 6), is that either capital L or lower-case l is allowed for the litre, in order to avoid possible confusion between the numeral 1 (one) and the lower-case letter l (el).

A multiple or sub-multiple prefix, if used, is part of the unit and precedes the unit symbol without a separator. A prefix is never used in isolation, and compound prefixes are never used.

Unit symbols are mathematical entities and not abbreviations. Therefore, they are not followed by a period except at the end of a sentence, and one must neither use the plural nor mix unit symbols and unit names within one expression, since names are not mathematical entities.

In forming products and quotients of unit symbols the normal rules of algebraic multiplication or division apply. Multiplication must be indicated by a space or a half-high (centred) dot ( $\cdot$ ), since otherwise some prefixes could be misinterpreted as a unit symbol. Division is indicated by a horizontal line, by a solidus (oblique stroke, /) or by negative exponents. When several unit symbols are combined, care should be taken to avoid ambiguities, for example by using brackets or negative exponents. A solidus must not be used more than once in a given expression without brackets to remove ambiguities.

It is not permissible to use abbreviations for unit symbols or unit names, such as sec (for either s or second), sq. mm (for either  $\text{mm}^2$  or square millimetre), cc (for either  $\text{cm}^3$  or cubic centimetre), or mps (for either m/s or metre per second). The use of the correct symbols for SI units, and for units in general, as listed in earlier chapters of this brochure, is mandatory. In this way ambiguities and misunderstandings in the values of quantities are avoided.

## 5.2 Unit names

Unit names are normally printed in upright type, and they are treated like ordinary nouns. In English, the names of units start with a lower-case letter (even when the symbol for the unit begins with a capital letter), except at the beginning of a sentence or in capitalized material such as a title. In keeping with this rule, the correct spelling of the name of the unit with the symbol °C is “degree Celsius” (the unit degree begins with a lower-case d and the modifier Celsius begins with an upper-case C because it is a proper name).

Although the values of quantities are normally expressed using symbols for numbers and symbols for units, if for some reason the unit name is more appropriate than the unit symbol, the unit name should be spelled out in full.

When the name of a unit is combined with the name of a multiple or sub-multiple prefix, no space or hyphen is used between the prefix name and the unit name. The combination of prefix name and unit name is a single word. See also Chapter 3.

When the name of a derived unit is formed from the names of individual units by multiplication, then either a space or a hyphen is used to separate the names of the individual units.

## 5.3 Rules and style conventions for expressing values of quantities

### 5.3.1 Value and numerical value of a quantity, and the use of quantity calculus

Symbols for quantities are generally single letters set in an italic font, although they may be qualified by further information in subscripts or superscripts or in brackets. For example,  $C$  is the recommended symbol for heat capacity,  $C_m$  for molar heat capacity,  $C_{m,p}$  for molar heat capacity at constant pressure, and  $C_{m,V}$  for molar heat capacity at constant volume.

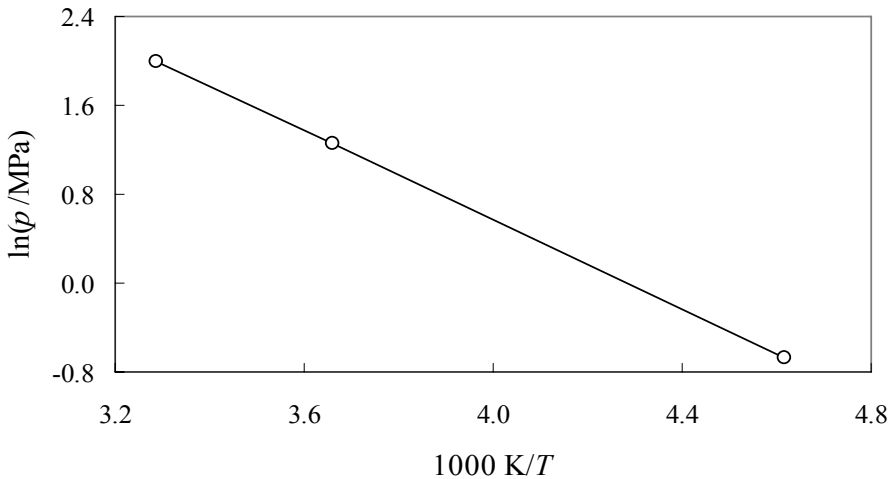
Recommended names and symbols for quantities are listed in many standard references, such as the ISO and IEC 80000 series *Quantities and units*, the IUPAP SUNAMCO Red Book *Symbols, Units and Nomenclature in Physics*, and the IUPAC Green Book *Quantities, Units and Symbols in Physical Chemistry*. However, symbols for quantities are recommendations (in contrast to symbols for units, for which the use of the correct form is mandatory). In particular circumstances authors may wish to use a symbol of their own choice for a quantity, for example in order to avoid a conflict arising from the use of the same symbol for two different quantities. In any such cases, the meaning of the symbol must be clearly stated. However, neither the name of a quantity, nor the symbol used to denote it, should imply any particular choice of unit.

Symbols for units are treated as mathematical entities. In expressing the value of a quantity as the product of a numerical value and a unit, both the numerical value and the unit may be treated by the ordinary rules of algebra. This procedure is described as the use of quantity calculus, or the algebra of quantities. For example, the equation  $T = 293 \text{ K}$  may equally be written  $T/\text{K} = 293$ . It is often convenient to write the quotient of a quantity and a unit in this way for the heading of a column in a table, so that the entries in the table are all simply numbers. For example, a table of vapour pressure against temperature, and the natural logarithm of vapour pressure against reciprocal temperature, may be formatted as shown below.



$T/\text{K}$	$10^3 \text{ K}/T$	$p/\text{MPa}$	$\ln(p/\text{MPa})$
216.55	4.6179	0.5180	-0.6578
273.15	3.6610	3.4853	1.2486
304.19	3.2874	7.3815	1.9990

The axes of a graph may also be labelled in this way, so that the tick marks are labelled only with numbers, as in the graph below.



Algebraically equivalent forms may be used in place of  $10^3 \text{ K}/T$ , such as  $\text{kK}/T$ , or  $10^3 (T/\text{K})^{-1}$ .

For example:  
The maximum electric potential difference is  $U_{\text{max}} = 1000 \text{ V}$  but not  $U = 1000 \text{ V}_{\text{max}}$ .  
The mass fraction of copper in the sample of silicon is  $w(\text{Cu}) = 1.3 \times 10^{-6}$  but not  $1.3 \times 10^{-6} w/w$ .

$m = 12.3 \text{ g}$  where  $m$  is used as a symbol for the quantity mass, but  $\varphi = 30^\circ 22' 8''$ , where  $\varphi$  is used as a symbol for the quantity plane angle.

$t = 30.2^\circ \text{C}$ ,  
but not  $t = 30.2^\circ \text{C}$ ,  
nor  $t = 30.2^\circ \text{C}$

a 10 k $\Omega$  resistor  
a 35-millimeter film

### 5.3.2 Quantity symbols and unit symbols

Unit symbols shall not be used to provide specific information about the quantity, and should never be the sole source of information on the quantity. Units are never qualified by further information about the nature of the quantity; any extra information on the nature of the quantity should be attached to the quantity symbol and not to the unit symbol.

### 5.3.3 Formatting the value of a quantity

The numerical value always precedes the unit, and a space is always used to separate the unit from the number. Thus the value of the quantity is the product of the number and the unit, the space being regarded as a multiplication sign (just as a space between units implies multiplication). The only exceptions to this rule are for the unit symbols for degree, minute, and second for plane angle,  $^\circ$ ,  $'$ , and  $''$ , respectively, for which no space is left between the numerical value and the unit symbol.

This rule means that the symbol  $^\circ \text{C}$  for the degree Celsius is preceded by a space when one expresses values of Celsius temperature  $t$ .

Even when the value of a quantity is used as an adjective, a space is left between the numerical value and the unit symbol. Only when the name of the unit is spelled out would the ordinary rules of grammar apply, so that in English a hyphen would be used to separate the number from the unit.

In any expression, only one unit is used. An exception to this rule is in expressing the values of time and of plane angles using non-SI units. However, for plane angles it is generally preferable to divide the degree decimally. Thus one would write 22.20° rather than 22° 12', except in fields such as navigation, cartography, astronomy, and in the measurement of very small angles.

$l = 10.234 \text{ m}$ , but not  
 $l = 10 \text{ m } 23.4 \text{ cm}$

### 5.3.4 Formatting numbers, and the decimal marker

The symbol used to separate the integral part of a number from its decimal part is called the decimal marker. Following the 22nd CGPM (2003, Resolution 10), the decimal marker “shall be either the point on the line or the comma on the line.” The decimal marker chosen should be that which is customary in the context concerned.

If the number is between +1 and –1, then the decimal marker is always preceded by a zero.

–0.234, but not –.234

Following the 9th CGPM (1948, Resolution 7) and the 22nd CGPM (2003, Resolution 10), for numbers with many digits the digits may be divided into groups of three by a thin space, in order to facilitate reading. Neither dots nor commas are inserted in the spaces between groups of three. However, when there are only four digits before or after the decimal marker, it is customary not to use a space to isolate a single digit. The practice of grouping digits in this way is a matter of choice; it is not always followed in certain specialized applications such as engineering drawings, financial statements, and scripts to be read by a computer.

43 279.168 29,  
 but not 43,279.168,29

either 3279.1683  
 or 3 279.168 3

For numbers in a table, the format used should not vary within one column.

### 5.3.5 Expressing the measurement uncertainty in the value of a quantity

The uncertainty that is associated with the estimated value of a quantity should be evaluated and expressed in accordance with the Guide JCGM 100:2008 (GUM 1995 with minor corrections), *Evaluation of measurement data - Guide to the expression of uncertainty in measurement*. The standard uncertainty associated with a quantity  $x$  is denoted by  $u(x)$ . One convenient way to represent the standard uncertainty is given in the following example:

$$m_n = 1.674\,927\,28\,(29) \times 10^{-27} \text{ kg},$$

where  $m_n$  is the symbol for the quantity (in this case the mass of a neutron), and the number in parentheses is the numerical value of the standard uncertainty of the estimated value of  $m_n$  referred to the last digit of the quoted value; in this case  $u(m_n) = 0.000\,000\,29 \times 10^{-27} \text{ kg}$ . If an expanded uncertainty  $U(x)$  is used in place of the standard uncertainty  $u(x)$ , then the coverage probability  $p$  and the coverage factor  $k$  must be stated.

### 5.3.6 Multiplying or dividing quantity symbols, the values of quantities, or numbers

When multiplying or dividing quantity symbols any of the following methods may be used:  $ab$ ,  $a\,b$ ,  $a \cdot b$ ,  $a \times b$ ,  $a/b$ ,  $\frac{a}{b}$ ,  $a\,b^{-1}$ .

When multiplying the value of quantities either a multiplication sign,  $\times$ , or brackets should be used, not a half-high (centred) dot. When multiplying numbers only the multiplication sign,  $\times$ , should be used.

Examples:

$F = ma$  for force equals  
 mass times acceleration

$(53 \text{ m/s}) \times 10.2 \text{ s}$   
 or  $(53 \text{ m/s})(10.2 \text{ s})$

$25 \times 60.5$   
 but not  $25 \cdot 60.5$

$(20 \text{ m})/(5 \text{ s}) = 4 \text{ m/s}$

$(a/b)/c$ , not  $a/b/c$

When dividing the values of quantities using a solidus, brackets are used to remove ambiguities.

### 5.3.7 Stating quantity values being pure numbers

As discussed in Section 2.2.3, values of quantities with unit one, are expressed simply as numbers. The unit symbol 1 or unit name “one” are not explicitly shown. Because SI prefix symbols can neither be attached to the symbol 1 nor to the name “one”, powers of 10 are used to express particularly large or small values.

Quantities that are ratios of quantities of the same kind (length ratios, amount fractions, etc.) have the option to be expressed with units (m/m, mol/mol) to aid the understanding of the quantity being expressed and also allow the use of SI prefixes, if this is desirable ( $\mu\text{m}/\text{m}$ ,  $\text{nmol}/\text{mol}$ ). Quantities relating to counting do not have this option, they are just numbers.

The internationally recognized symbol % (percent) may be used with the SI. When it is used, a space separates the number and the symbol %. The symbol % should be used rather than the name “percent”. In written text, however, the symbol % generally takes the meaning of “parts per hundred”. Phrases such as “percentage by mass”, “percentage by volume”, or “percentage by amount of substance” shall not be used; the extra information on the quantity should instead be conveyed in the description and symbol for the quantity.

The term “ppm”, meaning  $10^{-6}$  relative value, or 1 part in  $10^6$ , or parts per million, is also used. This is analogous to the meaning of percent as parts per hundred. The terms “parts per billion”, and “parts per trillion”, and their respective abbreviations “ppb”, and “ppt”, are also used, but their meanings are language dependent. For this reason the terms ppb and ppt shall be avoided.

$n = 1.51$ ,  
but not  $n = 1.51 \times 1$ ,  
where  $n$  is the quantity  
symbol for refractive  
index.

In English-speaking countries, a billion is now generally taken to be  $10^9$  and a trillion to be  $10^{12}$ ; however, a billion may still sometimes be interpreted as  $10^{12}$  and a trillion as  $10^{18}$ . The abbreviation ppt is also sometimes read as parts per thousand, adding further confusion.

### 5.3.8 Plane angles, solid angles and phase angles

The coherent SI unit for the plane angle and the phase angle is radian, unit symbol rad, and that for the solid angle is steradian, unit symbol sr.

The plane angle, expressed in radian, between two lines originating from a common point is the length of circular arc  $s$  swept out between the lines by a radius vector of length  $r$  from the common point divided by the length of the radius vector,  $\theta = s/r$  rad. The phase angle (often just referred to as the “phase”) is the argument of any complex number. It is the angle between the positive real axis and the radius of the polar representation of the complex number in the complex plane.

One radian corresponds to the angle for which  $s = r$ , thus  $1 \text{ rad} = 1$ . The measure of the right angle is exactly equal to the number  $\pi/2$ .

A historical convention is the degree. The conversion between radians and degrees follows from the relation  $360^\circ = 2\pi \text{ rad}$ . Note that the degree with the symbol “°” is not a unit of the SI.

The solid angle, expressed in steradian, corresponds to the ratio between an area  $A$  of the surface of a sphere of radius  $r$ , and the squared radius,  $\Omega = A/r^2$  sr. One steradian corresponds to the solid angle for which  $A = r^2$ , thus  $1 \text{ sr} = 1$ .

The units rad and sr correspond to ratios of two lengths and two squared lengths, respectively. However, it shall be emphasized that rad and sr must only be used to express angles and solid angles, but not to express ratios of lengths and squared lengths in general.

## Appendix 1. The Metre Convention and its Organs

On 20 May 1875 the Metre Convention was signed in Paris with the express purpose of assuring “*the international unification and perfection of the metric system*”. By this act, an intergovernmental organization was established with three organs:

- the General Conference on Weights and Measures (CGPM) made up of delegates of contracting Governments<sup>2</sup>,
- the International Committee for Weights and Measures (CIPM) placed under the authority of the CGPM and,
- the International Bureau of Weights and Measures (BIPM) that comes under the unique authority and direction of the CIPM.

The Convention was slightly modified in 1921, essentially to broaden the range of activities carried out under the Convention, but since then has remained unchanged. It continues to be the Convention through which contracting governments act together on matters related to measurement science and measurement standards. Today, this means the establishment and development of a uniform worldwide system of units, the International System of Units, SI.

The CIPM today has eighteen members each from a different state and elected at successive CGPMs. The president and secretary of the CIPM, under the terms of the Convention, present to governments an annual report on the administrative and financial position of the BIPM. The principal tasks of the CIPM remain the supervision and direction of the BIPM and the preparation of proposals for the General Conference and the implementation of decisions of the Conference. The CIPM meets annually at the Pavillon de Breteuil, a site conceded to the International Committee in perpetuity for this purpose by the French government in 1875. Reports of CIPM meetings are published by the BIPM in French and English.

Starting with the 1<sup>st</sup> CGPM in 1889, the 26<sup>th</sup> CGPM took place in 2018 and adopted the current version of the SI. Proceedings of CGPMs are published in French and English by the BIPM.

It is specified in the Metre Convention that all documents, proceedings and Resolutions of CGPMs as well as reports and publications of the CIPM and BIPM sent to member states must be in French. Today, all documents are published in English as well, although it is considered that the French versions are the official ones.

The BIPM, located on the site of the Pavillon de Breteuil, has a unique role based on its international and impartial character enabling it:

- to coordinate the realization and improvement of the world-wide measurement system to ensure it delivers accurate and comparable measurement results,
- to undertake selected scientific and technical activities that are more efficiently carried out in its own laboratories on behalf of member states, and
- to promote the importance of metrology to science, industry and society, in particular through collaboration with other intergovernmental organizations and international bodies and in international forums.

An important activity of the BIPM is to run the secretariat and maintain the Key Comparison Database (KCDB) of the CIPM *Mutual Recognition Arrangement for national*

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<sup>2</sup> Also referred to as states parties to the Convention or, informally, as member states.

*measurement standards and of calibration and measurement capabilities of national metrology institutes* (CIPM MRA). The CIPM MRA along with all other activities of the CIPM, its Consultative Committees and the BIPM is fully described on the BIPM website ([www.bipm.org](http://www.bipm.org)).

A series of Consultative Committees were created by the CIPM to advise it on matters related to the various areas of metrology following the extension of the range of work to be undertaken under the Metre Convention following its revision in 1921. Beginning with the Consultative Committee for Electricity in 1927, there are now a total ten Consultative Committees. In addition to electricity, these now cover the domains of photometry and radiometry (originally created in 1933), thermometry (1937), length (1952), time and frequency (1956), ionizing radiation (1958), mass and related quantities (1980), chemistry (1993) and acoustics (1999). The Consultative Committee for Units established in 1964 is charged with advising the CIPM on matters related to units and in particular with drawing up of this Brochure. The names of some of these Consultative Committees have changed since their original creation, the names given here are the current ones.

Each Consultative Committee is made up of member institutes or organizations chosen by the CIPM, which includes national metrology institutes and international organizations. The President of each committee is normally, but not necessarily, a member of the CIPM. Consultative Committees meet as necessary, usually once every two or three years.

## **Appendix 2. Historical notes on the development of the International System of Units and its base units.**

### **Part 1 The historical development of the realization of SI units.**

Experimental methods used for the realization of units using the equations of physics are known as primary methods. The essential characteristic of a primary method is that it allows a quantity to be measured in a particular unit directly from its definition by using only quantities and constants that themselves do not contain that unit.

Traditionally, a unit for a given quantity was taken to be a particular example of that quantity chosen to provide numerical values of common measurements of a convenient size. Before the rise of modern science, units were necessarily defined in terms of material artefacts, notably the metre and kilogram for length and mass, or the property of a particular object, namely the rotation of the earth for the second. Even at the origin of the metric system, however, at the end of the 18<sup>th</sup> century it was recognized that a more desirable definition of a unit of length for example would be one based on a universal property of nature such as the length of a pendulum beating seconds. Such a definition would be independent of time and place and in principle accessible all over the world. At the time, practical considerations resulted in the simpler, artefactual definitions for the metre and the kilogram and the second remained linked to the rotation of the Earth. It was only in 1960 that the first non-material definition was adopted namely the wavelength of a specified optical radiation for the metre.

Since then, definitions of the ampere, kelvin, mole and candela have been adopted that no longer referred to material artefacts but, in the case of the ampere to a specified electric current required to produce a given electromagnetic force, and in the case of the kelvin to a

particular thermodynamic state, the triple point of water. Even the atomic definition of the second was in terms of a specified transition of the atom of caesium. The kilogram has always stood out as the one unit that had resisted the transformation from an artefact. The definition that opened the way to real universality was that of the metre in 1983 which implied, although it did not state, a fixed numerical value for the speed of light. The definition was worded, however, in the traditional form and stated essentially that the metre was the distance travelled by light in a specified time. In this way it reflected the other definitions of the base units of the SI each of which has the same form, such as “the ampere is the current which...”, “the kelvin is a fraction of a specified temperature” and so on. Such definitions can be called explicit unit definitions.

Although they meet many of the requirements for universality and accessibility, and a variety of realizations are often possible, they nevertheless constrain practical realizations to experiments directly or indirectly linked to the particular conditions or states specified in the definitions. In consequence, the accuracy of realization of such definitions can never be better than the accuracy of realization of the particular conditions or states specified in the definitions.

This is a particular problem with the present definition of the second based on a microwave transition of an atom of caesium. Frequencies of optical transitions of different atoms or ions are now demonstrably more reproducible, by some orders of magnitude, than the defined frequency of caesium.

In the present definition of the SI based on the set of defining constants, instead of each definition specifying a particular condition or state, which sets a fundamental limit to the accuracy of realization, it is now open to us to choose any convenient equation of physics that links the particular constant or constants to the quantity we want to measure. This is a much more general way of defining the basic units of measurement. It is one that is not limited by today’s science or technology as future developments may lead to at present unknown equations that could result in quite different ways of realizing units to much higher accuracy. Defined in this way, there is, in principle, no limit to the accuracy with which a unit can be realized. The exception remains the definition of the second in which the original microwave transition of caesium remains, for the time being, the basis of the definition.

The difference between an explicit unit and an explicit constant definition can be clearly illustrated using the two previous definitions of the metre that depended upon a fixed numerical value of the speed of light and second the two definitions of the kelvin. The original 1983 definition of the metre states, in effect, that “the metre is the distance travelled by light in  $1/c$  seconds”. The new definition simply states that the metre is defined by taking the constant that defines the second, the specified caesium frequency and the fixed numerical value of the speed of light expressed in units  $\text{m}\cdot\text{s}^{-1}$ . We can thus use any equation of physics including, of course, that indicated by the former definition, the time taken to travel the given distance which is used for astronomical distances, but also the simple equation relating frequency and wavelength to the speed of light. The former definition of the kelvin based on a fixed numerical value for the temperature of the triple point of water requires ultimately a measurement at the triple point of water. The new definition, based on the fixed numerical value for the Boltzmann constant is clearly much more general in that any thermodynamic equation in which  $k$  appears can in principle be used to determine a thermodynamic temperature at any point on the temperature scale. For example, by determining the total radiant exitance of a black body at temperature  $T$ , equal to  $(2\pi^5 k^4/15c^2 h^3) T^4$ , in  $\text{Wm}^{-2}$  we can determine  $T$  directly.



For the kilogram, the unit whose definition has undergone the most fundamental change, realization can be through any equation of physics that links mass, the Planck constant, the velocity of light and the caesium frequency. One such equation is that which describes the operation of an electro-mechanical balance, known as a watt balance. With this a mechanical power, measured in terms of a mass,  $m$ , the acceleration due to gravity,  $g$ , and a velocity,  $v$ , can be measured in terms of an electrical power measured in terms of an electric current and voltage measured in terms of the quantum Hall and Josephson effects respectively. The resulting equation is  $mgv = Ch$  where  $C$  is a calibration constant that includes measured frequencies and  $h$  is Planck's constant.

Another method that can be used for a primary realization of the kilogram is through the determination of the number of atoms in a silicon sphere and using the equation:

$$m = \frac{8V2R_{\infty}h}{a_0^3 c \alpha^2 m_e} m_{Si}, \quad m = \frac{8V2R_{\infty}h}{a_0^3 c \alpha^2 m_e} m_{Si}$$

with the mass  $m$  and volume  $V$  of the sphere (about 1 kg), lattice parameter  $a_0$ , Rydberg constant  $R_{\infty}$ , fine structure constant  $\alpha$ , and the masses of a silicon atom (averaged over the three isotopes used for the sphere)  $m_{Si}$ , and the electron  $m_e$ , respectively. The first fraction corresponds to the number of atoms in the sphere, the second to the electron mass and the third fraction is the ratio of the mass of the (isotopically averaged) silicon atom the electron mass.

Another possibility for measuring mass through the new definition, but this time at the microscopic level, is through measurements of atomic recoil using the relation that includes  $h/m$ .

All these provide a striking illustration of the generality of the new way of defining units. Detailed information on the current realization of the base and other units is given on the BIPM website.

## Part 2 The historical development of the International System.

The 9th CGPM (1948, Resolution 6; CR 64) instructed the CIPM:

- to study the establishment of a complete set of rules for units of measurement;
- to find out for this purpose, by official enquiry, the opinion prevailing in scientific, technical and educational circles in all countries;
- to make recommendations on the establishment of a *practical system of units of measurement* suitable for adoption by all signatories to the *Convention of the Metre*.

The same CGPM also laid down, in Resolution 7 (CR 70), general principles for the writing of unit symbols, and listed some coherent derived units which were assigned special names.

The 10th CGPM (1954, Resolution 6; CR 80) adopted as base quantities and units for this practical system the following six quantities: length, mass, time, electric current, thermodynamic temperature, and luminous intensity, and the six corresponding base units: metre, kilogram, second, ampere, kelvin, and candela. After lengthy discussion between physicists and chemists, the 14th CGPM (1971, Resolution 3, CR 78, and *Metrologia* 1972, **8**, 36) added amount of substance, unit mole, as the seventh base quantity and unit.

The 11th CGPM (1960, Resolution 12; CR 87) adopted the name *Système international d'unités*, with the international abbreviation *SI*, for this practical system of units and laid

down rules for prefixes, derived units, and the former supplementary units, and other matters; it thus established a comprehensive specification for units of measurement. Subsequent meetings of the CGPM and the CIPM have added to and modified as necessary the original structure of the SI to take account of advances in science and of the new needs of users.

The historical sequence that led to these important decisions may be summarized as follows.

- The creation of the decimal metric system at the time of the French Revolution and the subsequent deposition of two platinum standards representing the metre and the kilogram, on 22 June 1799, in the *Archives de la République* in Paris, can be seen as the first step in the development of the present International System of Units.
- In 1832, Gauss strongly promoted the application of this metric system, together with the second defined in astronomy, as a coherent system of units for the physical sciences. Gauss was the first to make absolute measurements of the earth's magnetic field in terms of a decimal system based on the *three mechanical units* millimetre, gram and second for, respectively, the quantities length, mass, and time. In later years Gauss and Weber extended these measurements to include other electrical phenomena.
- These applications in the field of electricity and magnetism were further extended in the 1860s under the active leadership of Maxwell and Thompson through the British Association for the Advancement of Science (BAAS). They formulated the requirement for a *coherent system of units* with *base units* and *derived units*. In 1874 the BAAS introduced the *CGS system*, a three-dimensional coherent unit system based on the three mechanical units centimetre, gram and second, using prefixes ranging from micro to mega to express decimal submultiples and multiples. The subsequent development of physics as an experimental science was largely based on this system.
- The sizes of the coherent CGS units in the fields of electricity and magnetism proved to be inconvenient, so in the 1880s the BAAS and the International Electrical Congress, predecessor of the International Electrotechnical Commission (IEC), approved a mutually coherent set of *practical units*. Among them were the ohm for electrical resistance, the volt for electromotive force, and the ampere for electric current.
- After the signing of the Convention du Mètre on 20 May 1875, which created the BIPM and established the CGPM and the CIPM, work began on establishing new international prototypes for the metre and the kilogram. In 1889 the 1st CGPM sanctioned the international prototypes for the metre and the kilogram. Together with the astronomical second as the unit of time, these units constituted a three-dimensional mechanical unit system similar to the CGS system, but with the base units metre, kilogram, and second, the *MKS system*.
- In 1901 Giorgi showed that it is possible to combine the mechanical units of this MKS system with the practical electrical units to form a coherent four-dimensional system by adding to the three base units a fourth unit, of an electrical nature such as the ampere or the ohm, and also rewriting the equations occurring in electromagnetism in the so-called rationalized form. Giorgi's proposal opened the path to a number of new developments.

- After the revision of the Convention du Mètre by the 6th CGPM in 1921, which extended the scope and responsibilities of the BIPM to other fields in physics, and the subsequent creation of the Consultative Committee for Electricity (CCE) by the 7th CGPM in 1927, the Giorgi proposal was thoroughly discussed by the IEC, the International Union of Pure and Applied Physics (IUPAP), and other international organizations. This led the CCE to propose in 1939 the adoption of a four-dimensional system based on the metre, kilogram, second and ampere, the MKSA system, a proposal approved by the CIPM in 1946.
- Following an international enquiry by the BIPM, which began in 1948, the 10th CGPM, in 1954, approved the further introduction of the *kelvin*, and the *candela*, as base units for thermodynamic temperature and luminous intensity, respectively. The name *Système international d'unités*, with the abbreviation *SI*, was given to the system by the 11th CGPM in 1960. Rules for prefixes, derived units, and the former supplementary units, and other matters, were established, thus providing a comprehensive specification for all units of measurement.
- At the 14<sup>th</sup> CGPM in 1971 a new base unit the mole, symbol mol, was adopted for the quantity amount of substance. This followed a proposal from the International Standards Organization originating in a proposal from the SUN Commission of IUPAP, and supported by the International Union for Pure and Applied Chemistry (IUPAC). This brought to seven the number of base units of the SI. Since then, extraordinary advances have been made in relating SI units to truly invariant quantities such as the fundamental constants of physics and the properties of atoms. Recognizing the importance of linking SI units to such invariant quantities, the 24<sup>th</sup> CGPM, in 2011, adopted the principles of a new definition of the SI based on using a set of seven such constants as references for the definitions. At the time of the 24<sup>th</sup> CGPM, experiments to determine their values in terms of the then base units were not completely consistent but by the time of the 26<sup>th</sup> CGPM in 2018 this had been achieved and the new definition of the SI was adopted in Resolution 1. This is the basis of the definition presented in this brochure, and it is the simplest and most fundamental way of defining the SI.
- The SI was previously defined in terms of seven base units, and derived units defined as products of powers of the base units. The seven base units were chosen for historical reasons, as the metric system evolved and the SI developed over the last 130 years. Their choice was not unique, but it has become established and familiar over the years not only by providing a framework for describing the SI but also for defining the derived units. This role for the base units continues in the present SI even though the SI itself is now defined in terms of the seven defining constants. In this brochure therefore definitions of the seven base units will still be found but based on the seven defining constants, namely the caesium hyperfine frequency  $\Delta\nu_{\text{Cs}}$ , the speed of light in vacuum  $c$ , the Planck constant  $h$ , elementary charge  $e$ , Boltzmann constant  $k$ , Avogadro constant  $N_{\text{A}}$  and the luminous efficacy of a defined visible radiation  $K_{\text{cd}}$ .
- The definitions of the seven base units can be related unambiguously to the numerical values of the seven defining constants but there is not a one-to-one correspondence between the seven defining constants and the seven base units as many of the base units call upon more than one of the defining constants.

## Part 3 Historical perspective on the base units

### Unit of time, second

Before 1960, the unit of time the second, was defined as the fraction  $1/86\,400$  of the mean solar day. The exact definition of “mean solar day” was left to astronomers. However measurements showed that irregularities in the rotation of the Earth made this an unsatisfactory definition. In order to define the unit of time more precisely, the 11th CGPM (1960, Resolution 9, CR, 86) adopted a definition given by the International Astronomical Union based on the tropical year 1900. Experimental work, however, had already shown that an atomic standard of time, based on a transition between two energy levels of an atom or a molecule, could be realized and reproduced much more accurately. Considering that a very precise definition of the unit of time is indispensable for science and technology, the 13th CGPM (1967-1968, Resolution 1, CR, 103 and *Metrologia*, 1968, **4**, 43) chose a new definition of the second referenced to the frequency of the ground state hyperfine transition in the caesium 133 atom. A revised more precise wording of this same definition now in terms of a fixed numerical value of the unperturbed ground-state hyperfine splitting frequency of the caesium 133 atom,  $\Delta\nu_{\text{Cs}}$ , was adopted in Resolution 1 of the 26<sup>th</sup> CGPM in 2018.

### Unit of length, metre

The 1889 definition of the metre, namely, the length of the international prototype of platinum-iridium, was replaced by the 11th CGPM (1960) using a definition based on the wavelength of the radiation corresponding to a particular transition in krypton 86. This change was adopted in order to improve the accuracy with which the definition of the metre could be realized, this being achieved using an interferometer with a travelling microscope to measure the optical path difference as the fringes were counted. In turn, this was replaced in 1983 by the 17th CGPM (Resolution 1, CR, 97, and *Metrologia*, 1984, **20**, 25) with a definition referenced to the distance that light travels in vacuum in a specified interval of time, as presented in 2.4.2. The original international prototype of the metre, which was sanctioned by the 1st CGPM in 1889 (CR, 34-38), is still kept at the BIPM under conditions specified in 1889. In order to make clear its dependence on the fixed numerical value of the speed of light,  $c$ , the wording of the definition was changed in Resolution 1 of the 26<sup>th</sup> CGPM in 2018.

### Unit of mass, kilogram

The 1889 definition of the kilogram was simply the mass of the international prototype of the kilogram, an artefact made of platinum-iridium. This was, and still is, kept at the BIPM under the conditions specified by the 1st CGPM in 1889 (CR, 34-38) when it sanctioned the prototype and declared that “this prototype shall henceforth be considered to be the unit of mass”. Forty similar prototypes were made at about the same time, and these were all machined and polished to have closely the same mass as the international prototype. At the CGPM in 1889, after calibration against the international prototype, most of these were individually assigned to Member States of the Metre Convention, and some also to the

BIPM itself. The 3rd CGPM (1901, CR, 70), in a declaration intended to end the ambiguity in common usage concerning the use of the word “weight”, confirmed that “the kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram”. The complete version of these declarations appears on p. XXX.

By the time of the second verification of national prototypes in 1946, however, it was found that on average the masses of these prototypes were diverging from that of the international prototype. This was confirmed by the third verification from 1989 to 1991, the median difference being about 25 micrograms for the set of original prototypes sanctioned by the 1st CGPM in 1889. In order to assure the long-term stability of the unit of mass, to take full advantage of quantum electrical standards, and to be of more utility to modern science, it was therefore decided to adopt a new definition for the kilogram referenced to the value of a fundamental constant, for which purpose the Planck constant  $h$  was chosen, by Resolution 1 of the 26<sup>th</sup> CGPM 2018.

### **Unit of electric current, ampere**

Electric units, called “international units”, for current and resistance were introduced by the International Electrical Congress held in Chicago in 1893, and definitions of the “international ampere” and “international ohm” were confirmed by the International Conference in London in 1908.

It was already obvious on the occasion of the 8th CGPM (1933) that there was a unanimous desire to replace those “international units” by so-called “absolute units”. However because some laboratories had not yet completed experiments needed to determine the ratios between the international and absolute units the Conference gave authority to the CIPM to decide at an appropriate time both these ratios and the date at which the new absolute units would go into effect. This the CIPM did in 1946 (1946, Resolution 2, PV, 20, 129-137), when it decided that the new units would come into force on 1 January 1948. In October 1948 the 9th CGPM approved the decisions taken by the CIPM. The definition of the ampere chosen by the CIPM was referenced to the force between wires carrying an electric current, and it had the effect of fixing the value of the vacuum magnetic permeability  $\mu_0$  (also called magnetic constant). The value of the vacuum electric permittivity  $\epsilon_0$  (also called electric constant) then became fixed as a consequence of the new definition of the metre adopted in 1983.

However the 1948 definition of the ampere proved difficult to realize, and practical quantum standards (Josephson and quantum-Hall effects), which link the volt and the ohm to particular combinations of the Planck constant  $h$  and elementary charge  $e$ , have become almost universally used as a practical realization of the ampere through Ohm’s law (18th CGPM, 1987, Resolution 6, CR 100). As a consequence, it became natural not only to fix the numerical value of  $h$  to redefine the kilogram, but to fix the numerical value of  $e$  to redefine the ampere in order to bring the practical quantum electrical standards into exact agreement with the SI. The present definition based on a fixed numerical value for the elementary charge,  $e$ , was adopted in Resolution 1 of the 26<sup>th</sup> CGPM in 2018

### **Unit of thermodynamic temperature, kelvin**

The definition of the unit of thermodynamic temperature was given by the 10th CGPM (1954, Resolution 3; CR 79) which selected the triple point of water,  $T_{\text{TPW}}$ , as a fundamental fixed point and assigned to it the temperature 273.16 K, so defining the unit



kelvin. The 13th CGPM (1967-1968, Resolution 3; CR, 104 and *Metrologia*, 1968, **4**, 43) adopted the name kelvin, symbol K, instead of “degree kelvin”, symbol °K, for the unit defined in this way. However, the practical difficulties in realizing this definition, requiring a sample of pure water of well-defined isotopic composition, and the development of new primary methods of thermometry, led to the adoption of a new definition of the kelvin based on a fixed numerical value of the Boltzmann constant  $k$ . The present definition, which removed both of these constraints, was adopted in Resolution 1 of the 26<sup>th</sup> CGPM in 2018.

### Unit of amount of substance, mole

Following the discovery of the fundamental laws of chemistry, units called, for example, “gram-atom” and “gram molecule”, were used to specify amounts of chemical elements or compounds. These units had a direct connection with “atomic weights” and “molecular weights”, which are in fact relative atomic and molecular masses. “Atomic weights” were originally referred to the atomic weight of oxygen, by general agreement taken as 16. But whereas physicists separated the isotopes in a mass spectrometer and attributed the value 16 to one of the isotopes of oxygen, chemists attributed the same value to the (slightly variable) mixture of isotopes 16, 17 and 18, which was for them the naturally occurring element oxygen. Finally an agreement between the International Union of Pure and Applied Physics (IUPAP) and the International Union of Pure and Applied Chemistry (IUPAC) brought this duality to an end in 1959-1960. Physicists and chemists had agreed to assign the value 12, exactly, to the so-called atomic weight, correctly called the relative atomic mass  $A_r$ , of the isotope of carbon with mass number 12 (carbon 12,  $^{12}\text{C}$ ). The unified scale thus obtained gives the relative atomic and molecular masses, also known as the atomic and molecular weights, respectively. This agreement remained in force up until the 2018 redefinition”.

The quantity used by chemists to specify the amount of chemical elements or compounds is called “amount of substance”. Amount of substance, symbol  $n$ , is defined to be proportional to the number of specified elementary entities  $N$  in a sample, the proportionality constant being a universal constant which is the same for all entities. The proportionality constant is the reciprocal of the Avogadro constant  $N_A$ , so that  $n = N/N_A$ . The unit of amount of substance is called the *mole*, symbol mol. Following proposals by the IUPAP, the IUPAC, and the ISO, the CIPM gave a definition of the mole in 1967 and confirmed it in 1969, by specifying that the molar mass of carbon 12 should be exactly 0.012 kg/mol. This allowed the amount of substance  $n_S(X)$  of any pure sample  $S$  of entity  $X$  to be determined directly from the mass of the sample  $m_S$  and the molar mass  $M(X)$  of entity  $X$ , the molar mass being determined from its relative atomic mass  $A_r$  (atomic or molecular weight) without the need for a precise knowledge of the Avogadro constant, by using the relations

$$n_S(X) = m_S/M(X), \text{ and } M(X) = A_r(X) \text{ g/mol}$$

Thus, this definition of the mole was dependent on the artefact definition of the kilogram,

The numerical value of the Avogadro constant defined in this way was equal to the number of atoms in 12 grams of carbon 12. However, because of recent technological advances, this number is now known with such precision that a simpler and more universal definition of the mole has become possible, namely, by specifying exactly the number of entities in one mole of any substance, thus specifying exactly the value of the Avogadro constant. This has the effect that this new definition of the mole and the value of the Avogadro constant is no



longer dependent on the definition of the kilogram. Also the distinction between the fundamentally different quantities amount of substance and mass is thereby emphasized. The present definition of the mole based on a fixed numerical value for the Avogadro constant,  $N_A$ , was adopted in Resolution 1 of the 26<sup>th</sup> CGPM in 2018.

### **Unit of luminous intensity, candela**

The units of luminous intensity based on flame or incandescent filament standards in use in various countries before 1948 were replaced initially by the “new candle” based on the luminance of a Planckian radiator (a black body) at the temperature of freezing platinum. This modification had been prepared by the International Commission on Illumination (CIE) and by the CIPM before 1937, and the decision was promulgated by the CIPM in 1946. It was then ratified in 1948 by the 9th CGPM which adopted a new international name for this unit, the *candela*, symbol cd; in 1967 the 13th CGPM (Resolution 5, CR, 104 and *Metrologia*, 1968, **4**, 43-44) gave an amended version of this definition.

In 1979, because of the difficulties in realizing a Planck radiator at high temperatures, and the new possibilities offered by radiometry, i.e. the measurement of optical radiation power, the 16th CGPM (1979, Resolution 3, CR, 100 and *Metrologia*, 1980, **16**, 56) adopted a new definition of the candela.

The present definition of the candela is in terms of a fixed numerical value for the luminous efficacy of monochromatic radiation of frequency  $540 \times 10^{12}$  Hz,  $K_{cd}$ , adopted in Resolution 1 of the 26<sup>th</sup> CGPM in 2018.

*Supplementary content available on the BIPM website*

- Decisions of the CGPM and the CIPM*
- Practical realization of the definitions of some important units*
- Units for photochemical and photobiological quantities*